

Intermediate Algebra
Sample Final Exam Fall 2014

NAME KEY

You will have 2 hours to complete this exam. You may use a calculator (TI-84 or lower, no cell phones) but must show all algebraic work in the space provided to receive full credit. Read all directions carefully, simplify all answers fully, and clearly indicate your answer. Good Luck!

Factor Completely. If the polynomial is prime, say so. (2pts each)

1. $3x^2 + 14x + 8$

$$\begin{aligned} &3x^2 + 12x + 2x + 8 \\ &3x(x+4) + 2(x+4) \\ &\boxed{(x+4)(3x+2)} \end{aligned}$$

2. $5x^3 + 10x^2 - 15x$

GCF = $5x$

$$\begin{aligned} &5x[x^2 + 2x - 3] \\ &\boxed{5x(x+3)(x-1)} \end{aligned}$$

Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$, rationalize all denominators. (3 pts each)

3. $\frac{5x+5}{x-2} \cdot \frac{2x^2-8x+8}{x^2-1}$

$$\begin{aligned} &\frac{5(x+1) \cancel{2(x-2)}(x-2)}{(x-2) \cancel{(x+1)}(x-1)} \\ &\boxed{\frac{10(x-2)}{(x-1)}} \end{aligned}$$

4. $\frac{5x}{x-7} - \frac{3x}{7-x}$

$$\begin{aligned} &\frac{5x}{x-7} - \frac{-3x}{x-7} \\ &\boxed{\frac{8x}{x-7}} \end{aligned}$$

5. $\frac{x}{x^2+2x+1} + \frac{1}{x^2+5x+4}$

$$\begin{aligned} &\frac{x}{(x+1)(x+1)} + \frac{1}{(x+1)(x+4)} \\ &\frac{x(x+4) + 1(x+1)}{(x+1)(x+1)(x+4)} \\ &\boxed{\frac{x^2 + 5x + 1}{(x+1)(x+1)(x+4)}} \end{aligned}$$

6. $\sqrt[3]{54x^9y^7}$

$$\begin{aligned} &\sqrt[3]{54} \quad \sqrt[3]{x^9} \quad \sqrt[3]{y^7} \\ &3\sqrt[3]{2} \quad x^3 \quad y^2 \sqrt[3]{y} \\ &\boxed{3x^3y^2 \sqrt[3]{2y}} \end{aligned}$$

Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$, rationalize all denominators. (3pts each)

7. $\sqrt{\frac{5}{3}} = \frac{\sqrt{5}}{\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right)$

$$\boxed{\frac{\sqrt{15}}{3}}$$

8. $5\sqrt{12} + 6\sqrt{27}$

$$5 \cdot 2\sqrt{3} + 6 \cdot 3\sqrt{3}$$

$$10\sqrt{3} + 18\sqrt{3}$$

$$\boxed{28\sqrt{3}}$$

9. $(4 - \sqrt{5})(2 + \sqrt{5})$

$$8 + 4\sqrt{5} - 2\sqrt{5} - 5$$

$$\boxed{3 + 2\sqrt{5}}$$

10. $(3 - 2\sqrt{2}) - (5 - 4\sqrt{2})$

$$\boxed{-2 + 2\sqrt{2}}$$

11. $(3 - 2i)(4 - 3i)$

$$12 - 9i - 8i + 6i^2 - 6$$

$$\boxed{6 - 17i}$$

12. $\frac{10}{3+i} \left(\frac{3-i}{3-i} \right)$

$$\frac{30 - 10i}{9 - i^2 + 1} = \frac{30 - 10i}{10}$$

$$\boxed{3 - i}$$

Find the domain of each function below. Give your answer in interval notation. (2 pt each)

13. $f(x) = 2x^2 - 3x + 5$ - Polynomial

$\{x \mid \text{all real numbers}\}$

$(-\infty, \infty)$

14. $f(x) = \frac{2x-3}{x^2+7x+12}$ - Rational

$x^2 + 7x + 12 = 0$

$(x+3)(x+4) = 0$

$x = -3, x = -4$

$\{x \mid x \neq -3, x \neq -4\}$

$(-\infty, -4) \cup (-4, -3) \cup (-3, \infty)$

15. $f(x) = \sqrt{3x-5}$ - Radical

$3x - 5 \geq 0$

$3x \geq 5$

$x \geq \frac{5}{3}$

$\{x \mid x \geq \frac{5}{3}\}$

$[\frac{5}{3}, \infty)$

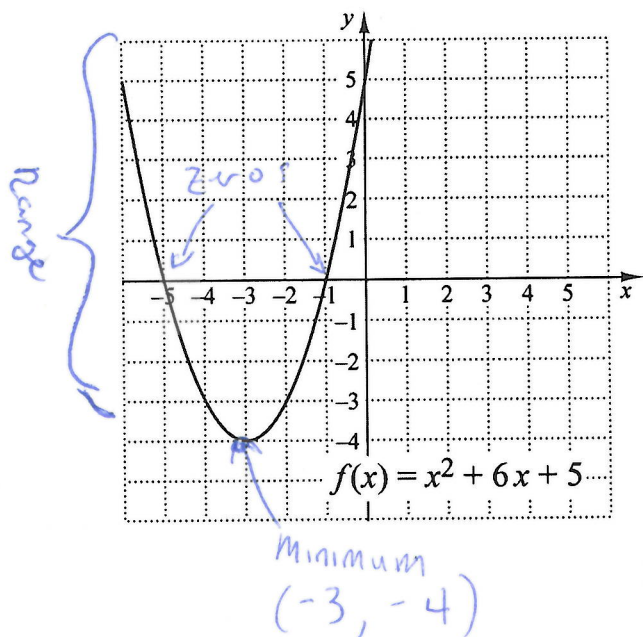
16. (1pt) Given $f(x) = x^2 - 3x + 5$, find $f(-3)$

$f(-3) = (-3)^2 - 3(-3) + 5$

$f(-3) = 9 + 9 + 5$

$f(-3) = 23$

17. Use the graph of the function below to determine the following: (2pts each)



What is the minimum value of the function? -4

What is the range of the function? $[-4, \infty)$

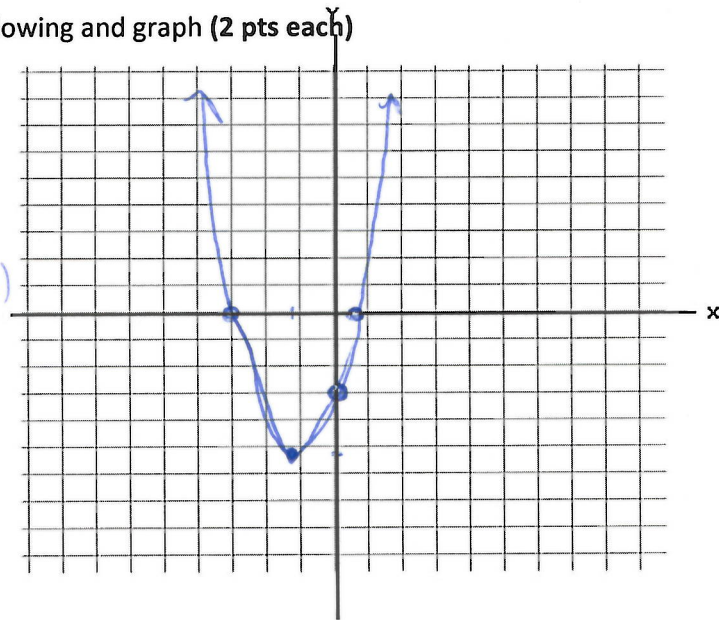
What are the zeros of the function? $x = -5$ and $x = -1$

18. For the quadratic function $f(x) = 2x^2 + 5x - 3$, find the following and graph (2 pts each)

Vertex ~~(-3, -4.5)~~ $(-\frac{5}{4}, -\frac{49}{8})$ $a=2, b=5, c=-3$
 $x = \frac{-b}{2a} = \frac{-5}{4}$

x-intercept(s) $(-3, 0)$ and $(\frac{1}{2}, 0)$ $f(-\frac{5}{4}) = -\frac{49}{8}$
 $y=0$ $0 = 2x^2 + 5x - 3$
 $0 = 2x^2 + 6x - x - 3$ $0 = 2(x+3) - 1(x+3)$
 $0 = (x+3)(2x-1)$
 $x = -3, x = \frac{1}{2}$

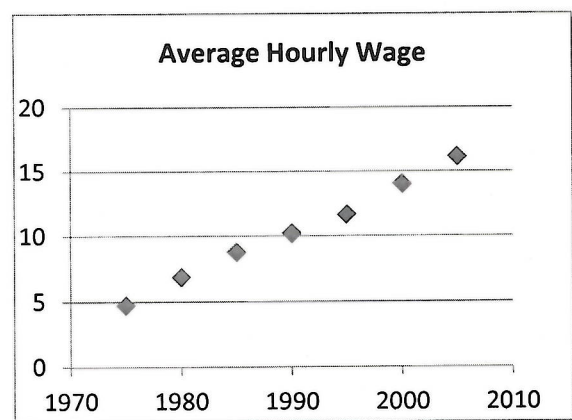
y-intercept $(0, -3)$
 $x=0$



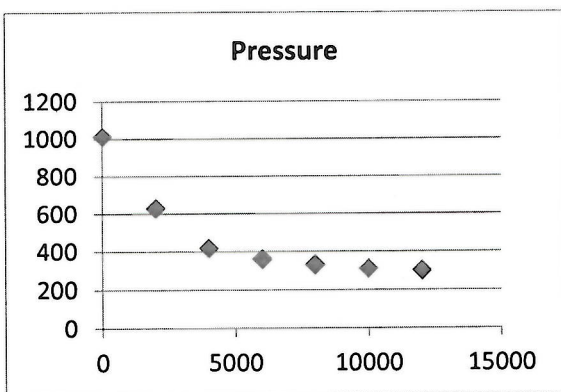
Match the graph to the type of function that best describes it. The same type may be used multiple times. (2pts each)

(a) : Linear (b) : Quadratic (c) : Exponential (d) : Radical

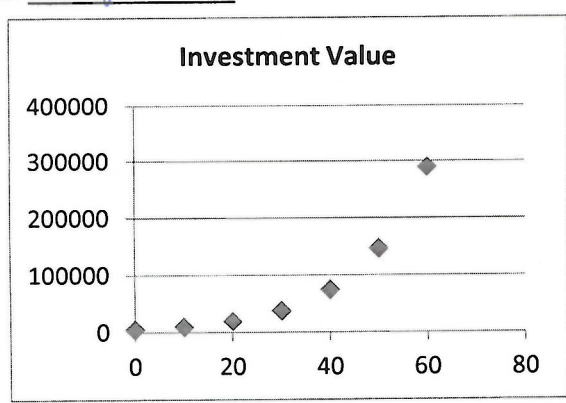
19. Linear



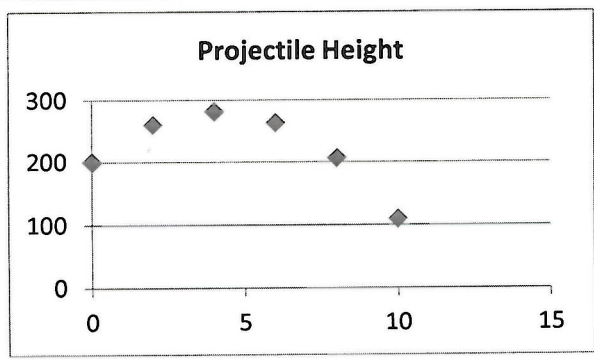
20. Exponential



21. Exponential



22. Quadratic



Solve each equation below. Simplify completely, do not round. (3 pts each)

23. $9x^2 + 3x = 0$ Factor

$$3x(3x+1) = 0$$

$$3x = 0, \quad 3x+1 = 0$$

$$3x = -1$$

$$x = 0, \quad x = -\frac{1}{3}$$

24. $\frac{50}{x-2} - \frac{16}{x} = \frac{30}{x}$

$$+ \frac{16}{x} \quad + \frac{16}{x}$$

$$\frac{50}{x-2} = \frac{46}{x} \quad \text{cross-multiply}$$

$$50x = 46(x-2)$$

$$50x = 46x - 92$$

$$-46x \quad -46x$$

$$4x = -92$$

$$\frac{4}{4} \quad \frac{-92}{4}$$

$$x = -23$$

25. $x = \sqrt{x-1} + 3$

$$(x-3)^2 = (\sqrt{x-1})^2 \quad \text{square}$$

$$x^2 - 6x + 9 = x - 1 \quad \text{factor}$$

$$-x + 1 \quad -x + 1$$

$$x^2 - 7x + 10 = 0$$

$$(x-2)(x-5) = 0$$

$$x = 2, \quad x = 5$$

↑
Doesn't
check

26. $x^2 + 10 = 6x$

quad. form.

$$x^2 - 6x + 10 = 0$$

$$a = 1$$

$$b = -6$$

$$c = 10$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{-4}}{2}$$

$$x = \frac{6 \pm 2i}{2}$$

$$x = 3 \pm i$$

27. $(\sqrt[3]{x-2})^3 = (-3)^3 \quad \text{cube}$

$$x-2 = -27$$

$$+2 \quad +2$$

$$x = -25$$

Application Problems. For all problems where an equation is not given, you need to define your variable(s), set up an algebraic equation or equations, solve algebraically, and answer the question with the proper units. If an equation is given, be sure to answer the question completely and with proper units. (4 pts each)

28. Suppose that a flare is launched upward with an initial velocity of 80 ft/sec from a height of 224 ft. Its height $h(t)$, in feet, after t seconds is given by

$$h(t) = -16t^2 + 80t + 224$$

After how long will the flare reach the ground?

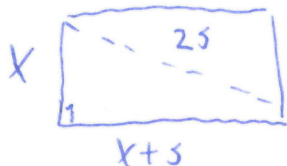
- ① time = ~~6~~
 ② $h(t) = 0$, $0 = -16t^2 + 80t + 224$
 $0 = -16[t^2 - 5t - 14]$
 ③ $0 = -16(t-7)(t+2)$
 $t = 7$ ~~$t = -2$~~

④ It will take 7 seconds

29. A rectangular garden is 5 feet longer than it is wide. Determine the dimensions of the garden if it measures 25 ft. diagonally.

- ① length = $x+5$
 width = x

② ~~length = 25~~



③ Use Pythagorean Thm

$$(x)^2 + (x+5)^2 = (25)^2$$

$$2x^2 + 10x + 25 = 625$$

$$2x^2 + 10x - 600 = 0$$

$$2(x^2 + 5x - 300) = 0$$

$$2(x+20)(x-15) = 0$$

$$x = \cancel{20} \quad x = 15$$

④ The garden is 15 feet by 20 feet.

30. The HP LaserJet works twice as fast as the Canon LaserJet. If the machines work together, a university can produce all its staff manuals in 15 hours. Find the time it would take each machine, working alone, to complete the same job.

- ① ~~HP~~ Machine Time = x
~~Canon~~ Machine Time = $2x$

② $\frac{15}{x} + \frac{15}{2x} = 1$

③ $\frac{30}{2x} + \frac{15}{2x} = 1$
 $\frac{45}{2x} = 1$
 $45 = 2x$

$$\frac{45}{2} = \frac{2x}{2}$$

$$22.5 = x$$

④ HP machine alone takes 22.5 hrs
 Canon machine alone takes 45 hrs

31. A plane travels 120mph in still air and flies 140 miles into the wind and 140 miles with the wind in a total of 2.4 hours. Find the wind speed.

① wind speed = x

②

	d	r	t
with wind	140	$120 + x$	$\frac{140}{120 + x}$
into wind	140	$120 - x$	$\frac{140}{120 - x}$

③ Total Time = 2.4

$$\frac{140}{120 + x} + \frac{140}{120 - x} = 2.4$$

$$140(120 - x) + 140(120 + x) = 2.4(120 + x)(120 - x)$$

$$33600 = -2.4x^2 + 34560$$

$$2.4x^2 - 960 = 0$$

$$2.4(x^2 - 400) = 0$$

$$2.4(x + 20)(x - 20) = 0$$

$$x = -20 \quad x = 20$$

④ The wind speed is 20 mph

32. The average 140-lb adult burns about 160 calories playing touch football for 20 minutes. How long would the average 140-lb adult need to play in order to burn 200 calories?

① how long = x

② $\frac{\text{Time}}{\text{Calories}} = \frac{20}{160} = \frac{x}{200}$

③ $4000 = 160x$
 $25 = x$

④ It will take 25 minutes

33. Ted invests \$6250 in account that earns interest and is compounded annually. After 2 years his investment has grown to \$6760. What was the interest rate on the account?

① rate = r

② $A = P(1 + r)^t$

③ $\frac{6760}{6250} = \frac{6250}{6250}(1 + r)^2$
 $1.0816 = (1 + r)^2$
 $1.04 = 1 + r$
 $-1 \quad -1$

$$0.04 = r$$

$$4\% = r$$

④ The Interest rate was 4%