Intermediate Algebra
Fall 2015 Sample Final Answer Key

You will have 2 hours to complete this exam. You may use a calculator (TI-84 or lower, no cell phones) but must show all algebraic work in the space provided to receive full credit. Read all directions carefully, simplify all answers fully, and clearly indicate your answer. Good Luck!

Factor Completely. If the polynomial is prime, say so. (2pts each)

1. $15x^2 - 14x + 3$

$15x^2 - 9x - 5x + 3$

$3x(5x-3) - 1(5x-3)$

$\boxed{(5x-3)(3x-1)}$

2. $27x^3 - 3x$

$3x(9x^2 - 1)$

$\boxed{3x(3x+1)(3x-1)}$

Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$, rationalize all denominators. (3 pts each)

3. $\frac{x^2-1}{5x-2} \cdot \frac{15x-6}{x^2+5x-6}$

$\frac{(x+1)(x-1)}{(5x-2)} \cdot \frac{3(5x-2)}{(x+6)(x-1)} = \boxed{\frac{3(x+1)}{(x+6)}}$

4. $\frac{x^2}{x-3} + \frac{9}{3-x}$

$\frac{x^2}{x-3} + \frac{-9}{x-3} = \frac{x^2-9}{x-3} = \frac{(x+3)(x-3)}{(x-3)} = \boxed{x+3}$

Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$, rationalize all denominators. (3 pts each)

5. $\dfrac{x}{x^2+9x+20} - \dfrac{4}{x^2+7x+12}$

$\dfrac{x}{(x+4)(x+5)} + \dfrac{-4}{(x+4)(x+3)}$ LCD $= (x+3)(x+4)(x+5)$

$\dfrac{x(x+3)}{(x+3)(x+4)(x+5)} + \dfrac{-4(x+5)}{(x+3)(x+4)(x+5)} = \dfrac{x^2+3x-4x-20}{(x+3)(x+4)(x+5)}$

$= \dfrac{x^2-x-20}{(x+3)(x+4)(x+5)} = \dfrac{(x-5)(x+4)}{(x+3)(x+4)(x+5)} = \boxed{\dfrac{x-5}{(x+3)(x+5)}}$

6. $\sqrt[4]{81x^7y^4}$

$\sqrt[4]{3^4x^7y^4} = \boxed{3xy\sqrt[4]{x^3}}$

7. $\sqrt{\dfrac{36}{7}} = \dfrac{\sqrt{36}}{\sqrt{7}} = \dfrac{6}{\sqrt{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}} = \boxed{\dfrac{6\sqrt{7}}{7}}$

8. $5\sqrt{12} + 4\sqrt{27}$

$5\sqrt{2^2 \cdot 3} + 4\sqrt{3^3} = 10\sqrt{3} + 12\sqrt{3}$

$= \boxed{22\sqrt{3}}$

Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$, rationalize all denominators. (3pts each)

9. $(4 - \sqrt{3})(5 - \sqrt{3})$

$20 - 4\sqrt{3} - 5\sqrt{3} + 3$

$\boxed{23 - 9\sqrt{3}}$

10. $(2 - 3\sqrt{2}) - (5 - 5\sqrt{2})$

$2 - 3\sqrt{2} + -5 + 5\sqrt{2}$

$\boxed{-3 + 2\sqrt{2}}$

11. $(2 + 3i)(4 - 3i)$

$8 - 6i + 12i - 9i^2$

$8 + 6i + 9$

$\boxed{17 + 6i}$

12. $\frac{26}{5+i}$

$\frac{26}{5+i} \cdot \frac{(5-i)}{(5-i)} = \frac{130 - 26i}{25 - 5i + 5i - i^2} = \frac{130 - 26i}{25 + 1}$

$= \frac{130 - 26i}{26} = \frac{130}{26} - \frac{26}{26}i$

$= \boxed{5 - i}$

List any restrictions on the domain of each function below (1pt each)

13. $f(x) = x^2 - 5x - 14$

Domain = All $\mathbb{R}$ or $(-\infty, \infty)$

No restrictions on domain

14. $f(x) = \frac{2x-3}{x^2-3x+2}$

$x^2 - 3x + 2 \neq 0$

$(x-2)(x-1) \neq 0$

$x \neq 2$ or $x \neq 1$

Restrictions: $x \neq 2, x \neq 1$

15. $f(x) = \sqrt{4x-5}$

Domain $4x - 5 \geq 0$

$4x \geq 5$

$x \geq 5/4$

Domain $x \geq 5/4$ or $[5/4, \infty]$

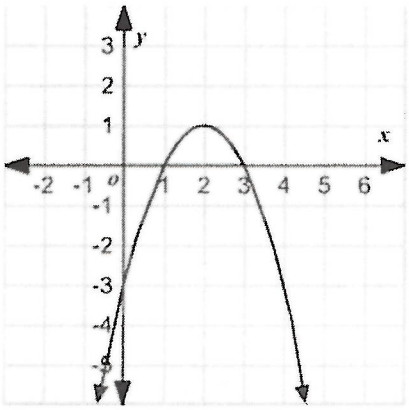
Restrictions: $x < 5/4$ or $(-\infty, 5/4)$

16. **(2pts)** Given $f(x) = x^2 - 7x + 15$, find $f(-3)$

$(-3)^2 - 7(-3) + 15$

$9 + 21 + 15 = 45$

17. Use the graph of the function below to determine the following: **(2pts each)**



What is the maximum value of the function? 1

What is the range of the function? $(-\infty, 1]$

or $y \leq 1$

What are the zeros of the function? 1 and 3

18. For the quadratic function $f(x) = x^2 - 2x - 8$, find the following and graph **(2pts each)**

a. Vertex $(1, -9)$

$x = -b/2a = 2/2 = 1$

$y = f(1) = 1^2 - 2(1) - 8 = -9$

b. x-intercept(s) $(4, 0), (-2, 0)$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

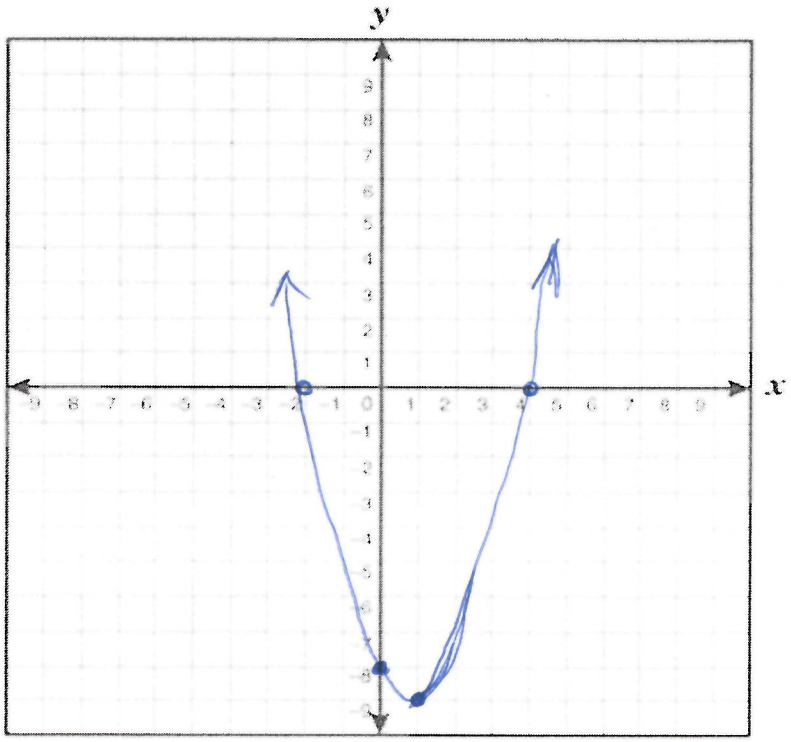
$x = 4$ or $x = -2$

c. y-intercept $(0, -8)$

$f(0) = 0^2 - 2(0) - 8$

$= -8$

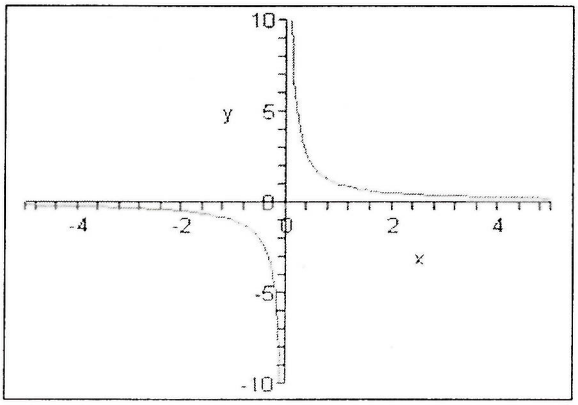
d. Graph to the right



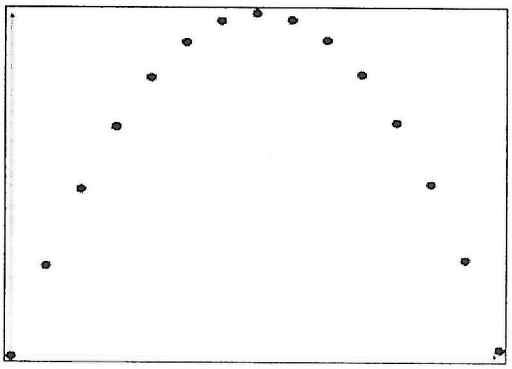
Match the graph to the type of function that best describes it. The same type may be used multiple times or not at all. (2pts each)

(a) : Linear (b) : Quadratic (c) : Exponential (d) : Radical (e): Rational

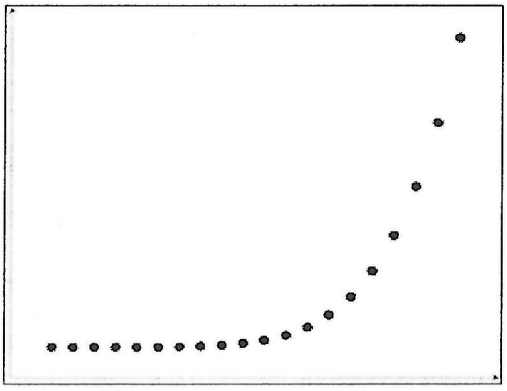
19. Rational



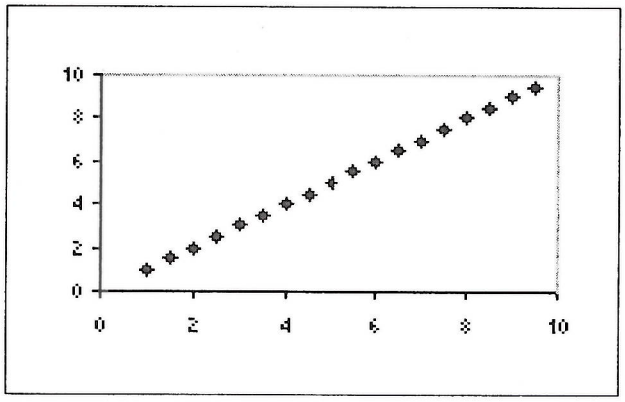
20. Quadratic



21. Exponential



22. Linear



Solve each equation below. Simplify completely, do not round. (3 pts each)

23. $3x - 12x^2 = 0$

$3x(1-4x)=0$

$3x=0$ or $1-4x=0$

$x=0$ or $x=1/4$

24. $\frac{x+3}{x+2} - \frac{x}{x^2-4} = \frac{x}{x-2}$

$\frac{(x+2)(x-2)}{1}\frac{x+3}{(x+2)} + \frac{-x\,(x+2)(x-2)}{(x+2)(x-2)} = \frac{x\,(x+2)(x-2)}{(x-2)}$ LCD $=(x+2)(x-2)$

$(x+3)(x-2) + -x = x(x+2)$

$x^2-2x+3x-6-x = x^2+2x$

$-6 = 2x$

$-3 = x$

25. $3 + \sqrt{x-1} = x$

$(\sqrt{x-1})^2 = (x-3)^2$

$x-1 = x^2-6x+9$

$0 = x^2-7x+10$

$0 = (x-2)(x-5)$

$x=2$ or $x=5$

check: $x=2$

$3+\sqrt{2-1} = 2$

$3+1=2$ X

check: $x=5$

$3+\sqrt{5-1} = 5$

$3+2=5$ ✓

only solution

Solve each equation below. Simplify completely, do not round. (3 pts each)

26. $x^2 + 10 = -6x$

$x^2 + 6x + 10 = 0$ $\quad a = 1, b = 6, c = 10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{-4}}{2}$$

$$x = \frac{-6 \pm 2i}{2} = \boxed{-3 \pm i}$$

27. $\left(\sqrt[3]{3 - x}\right)^3 = (-3)^3$

$3 - x = -27$

$-x = -30$

$\boxed{x = 30}$

Application Problems. For all problems where an equation is not given, you need to define your variable(s), set up an algebraic equation or equations, solve algebraically, and answer the question with the proper units. If an equation is given, be sure to answer the question completely and with proper units. (4 pts each)

28. Suppose that a flare is launched upward with an initial velocity of 80 ft/sec from a height of 224 ft. Its height, $h(t)$, in feet, after t seconds is given by

$$h(t) = -16t^2 + 80t + 224$$

After how long with the flare reach the ground?

$-16t^2 + 80t + 224 = 0$

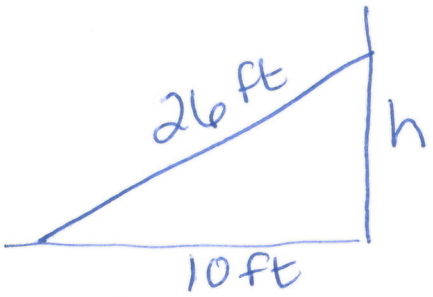
$-16(t^2 - 5t - 14) = 0$

$-16(t-7)(t+2) = 0$

$t = 7$ or ~~$t = -2$~~

after 7 sec

29. A 26-foot ladder is leaning against a house. If the bottom of the ladder is 10 feet from the base of the house, how high does the ladder reach?



$h^2 + 10^2 = 26^2$

$h^2 + 100 = 676$

$h^2 = 576$

$\sqrt{h^2} = \pm\sqrt{576}$

$h = \pm 24$

24 feet

30. Jean-Marc can correct a set of final exams twice as fast as Sue can. If they work together they can correct a set of exams in 2 hours. How long would it take each of them working alone to correct that set of exams?

x = time for JM alone

$2x$ = time for Sue alone

$$\frac{2x}{1}\cdot\frac{2}{x}+\frac{2}{2x}\cdot\frac{2x}{1}=1\cdot 2x$$

$$4+2=2x$$

$$6=2x$$

$$3=x$$

JM alone
3 hours
Sue alone
6 hours

31. Mike's boat travels 15 km/hr in still water. He travels 140 km downstream in the same time that it takes to travel 35 km upstream. What is the speed of the river?

$D = r \cdot t$

downstream	140	$15+r$	t	$t=\frac{140}{15+r}$
upstream	35	$15-r$	t	$t=\frac{35}{15-r}$

$$\frac{140}{15+r}=\frac{35}{15-r}$$

$$140(15-r)=35(15+r)$$

$$2100-140r=525+35r$$

$$1575=175r$$

$$9=r$$

speed of river
9 km/hr

32. Nick's Sporting Goods Store makes exercise machines. Nick has determined that when x hundred exercise machines are produced, the average cost per machine can be estimated by

$C(x) = 0.1x^2 - 0.7x + 2.425$ where $C(x)$ is in hundreds of dollars. What is the minimum average cost per machine and how many exercise machines should be built in order to achieve that minimum?

Min at vertex

$x = \frac{-b}{2a} = \frac{.7}{2(.1)} = 3.5 \quad \times 100 \quad$ 350 exercise machines

$C(3.5) = .1(3.5)^2 - .7(3.5) + 2.425$

$= 1.2 \times 100 = \$120$

Min Cost/machine = \$120 @ 350 machines

33. Hendree invests \$ 6250 in account that earns interest and is compounded annually. After 2 years his investment has grown to \$6760. What was the interest rate on the account?

$6760 = 6250(1+r)^2$

$\frac{6760}{6250} = (1+r)^2$

$\pm\sqrt{1.0816} = \sqrt{(1+r)^2}$

$\pm 1.04 = 1 + r$

$r = .04$ or -2.04

4%

Bonus (3pts): What positive integer is four times its square root?

$$(x)^2 = (4\sqrt{x})^2$$

$$x^2 = 16x$$

$$x^2 - 16x = 0$$

$$x(x-16) = 0$$

$x \neq 0$ or $x = 16$

Have a wonderful summer!