Intermediate Algebra
Sample Final Exam Fall 2016

You will have 2 hours to complete this exam. You may use a calculator (TI-84 or lower, no cell phones) but must show all algebraic work in the space provided to receive full credit. Read all directions carefully, simplify all answers fully, and clearly indicate your answer. Good Luck!

Factor Completely. If the polynomial is prime, say so. (2pts each)

1. $9x^2 - x - 8$

$9x^2 - 9x + 8x - 8$

$9x(x-1) + 8(x-1)$

$\boxed{(9x+8)(x-1)}$

2. $49 - 9x^2$

$\boxed{(7+3x)(7-3x)}$

Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$, rationalize all denominators. (3 pts each)

3. $\frac{8x+8}{2x^2+x-1} \cdot \frac{x^2-1}{x^2-2x+1}$

$\frac{8(x+1)}{(2x-1)(x+1)} \cdot \frac{(x+1)(x-1)}{(x-1)(x-1)} = \boxed{\frac{8(x+1)}{(2x-1)(x-1)}}$

4. $\frac{4x^2}{2x-5} + \frac{25}{5-2x} = \frac{4x^2}{2x-5} + \frac{-25}{2x-5}$

$= \frac{4x^2-25}{2x-5} = \frac{(2x+5)(2x-5)}{(2x-5)} = \boxed{(2x+5)}$

Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$, rationalize all denominators. (3 pts each)

5. $\frac{6}{x^2+x-2} + \frac{4}{x^2-4x+3}$

$(x+2)(x-1)$ $(x-3)(x-1)$

$$\frac{6(x-3)}{(x-1)(x+2)(x-3)} + \frac{4(x+2)}{(x-1)(x+2)(x-3)}$$

$$\frac{6x-18+4x+8}{(x-1)(x+2)(x-3)} = \frac{10x-10}{(x-1)(x+2)(x-3)} = \frac{10(x-1)}{(x-1)(x+2)(x-3)}$$

$$= \boxed{\frac{10}{(x+2)(x-3)}}$$

6. $\sqrt[4]{16x^8y^5}$

$$\sqrt[4]{2^4x^8y^4y} = \boxed{2x^2y\sqrt[4]{y}}$$

7. $\sqrt{\frac{25}{3}}$ $\frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{5\sqrt{3}}{3}}$

8. $2\sqrt{75} - 9\sqrt{3}$

$2\sqrt{5^2 \cdot 3} - 9\sqrt{3}$

$10\sqrt{3} - 9\sqrt{3} = \boxed{\sqrt{3}}$

Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$, rationalize all denominators. (3pts each)

9. $(3 + \sqrt{5})(2 - \sqrt{5})$

$6 - 3\sqrt{5} + 2\sqrt{5} - 5$

$\boxed{1 - \sqrt{5}}$

10. $(4 - 3\sqrt{2}) - (1 - 4\sqrt{2})$

$4 - 3\sqrt{2} - 1 + 4\sqrt{2}$

$\boxed{3 + \sqrt{2}}$

11. $(2 + 3i)(4 - 3i)$

$8 - 6i + 12i - 9i^2$

$8 + 6i + 9$

$\boxed{17 + 6i}$

12. $\frac{26}{5+i}\left(\frac{5-i}{5-i}\right)$

$\frac{130 - 26i}{25 - 5i + 5i - i^2} = \frac{130 - 26i}{26}$

$= \boxed{5 - i}$

List any restrictions on the domain of each function below (1pt each)

13. $f(x) = 3x^2 - 4x - 15$

no restrictions

14. $f(x) = \frac{x+5}{x^2-x-20}$

$x^2 - x - 20 = 0$

$(x-5)(x+4) = 0$

$x = 5$ or $x = -4$

restrictions at $x=5, x=-4$

15. $f(x) = \sqrt{4-x}$

$4 - x < 0$

$-x < -4$

$x > 4$

restrictions $x > 4$

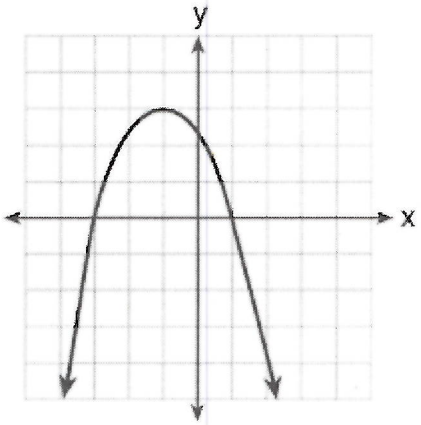
16. **(2pts)** Given $f(x) = 2x^2 - 5x + 1$, find $f(-2)$

$2(-2)^2 - 5(-2) + 1$

$8 + 10 + 1$

19

17. Use the graph of the function below to determine the following: **(2pts each)**



What is the maximum value of the function? 3

What is the range of the function? $(-\infty, 3]$

What are the zeros of the function? -3 and 1

18. For the quadratic function $f(x) = x^2 - 6x + 5$, find the following and graph **(2pts each)**

a. Vertex (3, -4)

$x = \frac{6}{2} = 3$

$f(3) = 9 - 18 + 5 = -4$

b. x-intercept(s) (5, 0) (1, 0)

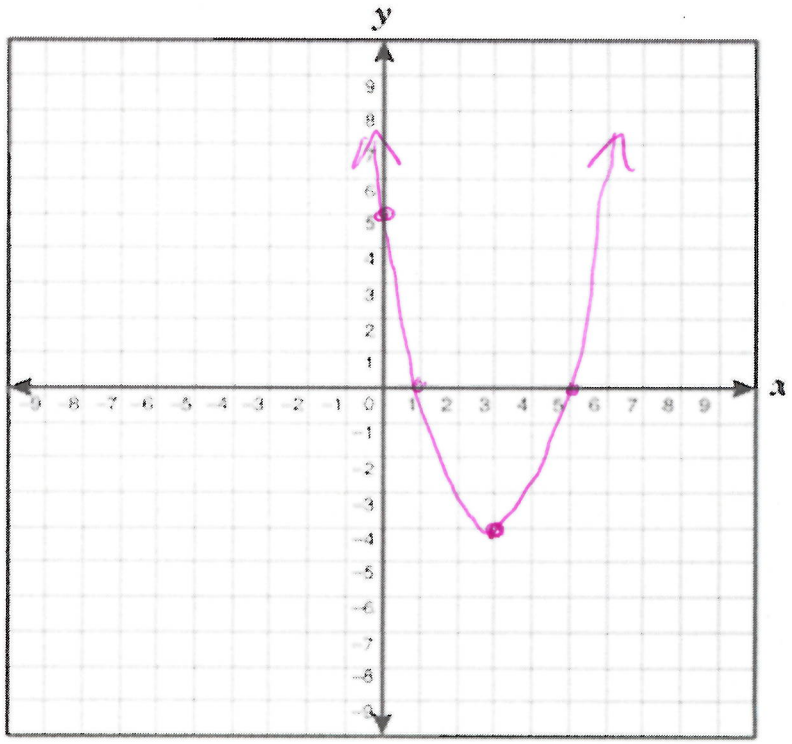
$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$x = 5$ or $x = 1$

c. y-intercept (0, 5)

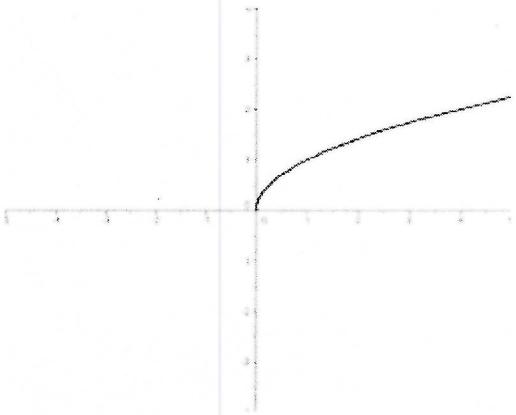
d. Graph to the right



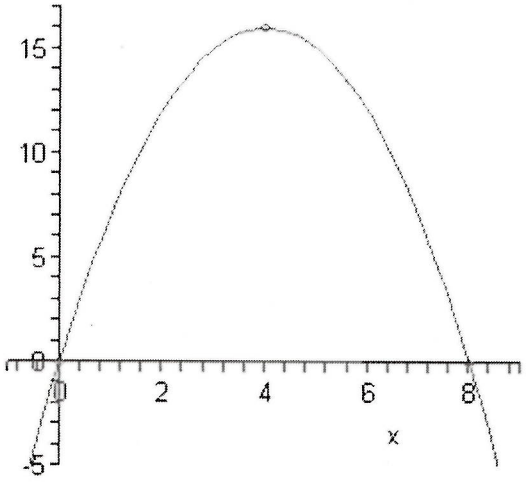
Match the graph to the type of function that best describes it. The same type may be used multiple times or not at all. (2pts each)

(a) : Linear (b) : Quadratic (c) : Exponential (d) : Radical (e): Rational

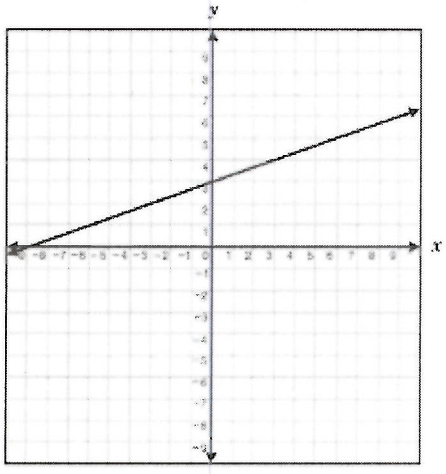
19. D Radical



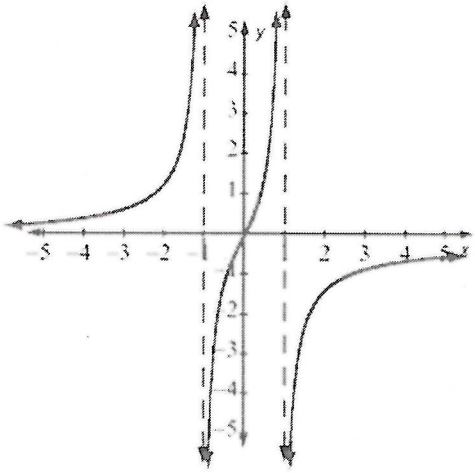
20. B Quadratic



21. A Linear



22. E Rational



Solve each equation below. Simplify completely, do not round. (3 pts each)

23. $6x^2 - 7x = 3$

$6x^2 - 7x - 3 = 0$

$6x^2 - 9x + 2x - 3 = 0$

$3x(2x-3) + 1(2x-3) = 0$

$(3x+1)(2x-3) = 0$

$3x+1=0$ or $2x-3=0$

$x = -1/3$ or $x = 3/2$

24. $x = \sqrt{x-1} + 3$

$-3 \quad -3$

$(x-3)^2 = (\sqrt{x-1})^2$

$x^2 - 6x + 9 = x - 1$

$x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

$x-5=0$ or $x-2=0$

$x = 5$ ~~$x = 2$~~

$5 = \sqrt{4} + 3$

$5 = 5$ ✓

$2 = \sqrt{1} + 3$

$2 = 4$ X

25. $\frac{2}{x-2} + \frac{1}{x+4} = \frac{x}{x^2+2x-8}$

$(x+4)(x-2)$

$2(x+4) + 1(x-2) = x$

$2x + 8 + x - 2 = x$

$3x + 6 = x$

$2x = -6$

$x = -3$

Solve each equation below. Simplify completely, do not round. (3 pts each)

26. $\left(\sqrt[3]{2x-1}\right)^3 = (-2)^3$

$2x - 1 = -8$

$2x = -7$

$\boxed{x = -7/2}$

27. $x^2 + 6x + 34 = 0$

$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(34)}}{2(1)}$

$x = \dfrac{-6 \pm \sqrt{-100}}{2}$

$x = \dfrac{-6 \pm 10i}{2}$

$\boxed{x = -3 \pm 5i}$

Application Problems. For all problems where an equation is not given, you need to define your variable(s), set up an algebraic equation or equations, solve algebraically, and answer the question with the proper units. If an equation is given, be sure to answer the question completely and with proper units. (4 pts each)

28. For many people suffering from constricted bronchial muscles, the drug Albuterol is prescribed. The number of micrograms A of Albuterol in a person's bloodstream t minutes after 200 micrograms have been inhaled can be approximated by $A = -50t^2 + 200t$. How long after inhalation will there be about 150 micrograms of Albuterol in the bloodstream?

$150 = -50t^2 + 200t$

$50t^2 - 200t + 150 = 0$

$50(t^2 - 4t + 3) = 0$

$50(t-3)(t-1) = 0$

$t - 3 = 0$ or $t - 1 = 0$

$t = 3$ or $t = 1$

after 1 sec and after 3 sec

29. The bookstore wants to maximize its profit from the sale of TI-30 calculators. Their Profit function can be approximated by $P(x) = -150x^2 + 900x + 600$, where $P(x)$ is the profit in dollars from selling x hundred calculators. How many calculators should they sell to maximize their profit and what will the profit be?

Max at vertex

$x = \frac{-900}{2(-150)} = 3$

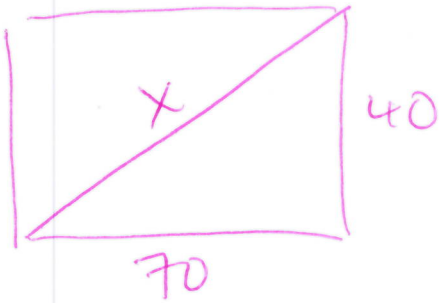
$P(3) = -150(3)^2 + 900(3) + 600$

$P(3) = 1950$

Number of Calculators 300

Max Profit $1950

30. Students at Mathway Community College have worn a path that cuts diagonally across the campus "quad". The quad is a rectangle that measures 70 yards by 40 yards. What is the length of the diagonal path? How many yards does a student save by using the diagonal path instead of going around the edge of the quad to get across? Round answers to the nearest tenth of a yard



$40^2 + 70^2 = x^2$

$\sqrt{6500} = \sqrt{x^2}$

$80.6 \approx x$

$70 + 40 = 110$

$110 - 80.6 = 29.4$

Length of diagonal path 80.6 yds

Yards saved 29.4 yds

31. Amtrak offers both a high-speed train and a regular train. The high speed train can travel at speed that is 50 mph faster than the regular train. In the same time that it takes the high speed train to travel 276 miles, the regular train travels 156 miles. What is the speed of each train?

$D = r \cdot t$

High speed	276	$r+50$	$\frac{276}{r+50}$
Reg. Train	156	r	$\frac{156}{r}$

$\frac{276}{r+50} = \frac{156}{r}$

$276r = 156(r+50)$

$276r = 150r + 7800$

$120r = 7800$

$r = 65$

Speed of regular train 65 mph

Speed of High-speed train 115 mph

32. Nick can refinish the floor in an apartment twice as fast as Mike. If they work together it will take them 4 hours to refinish the floor. How long would it take each of them if they worked alone?

x = time for Nick

$2x$ = time for Mike

$$\left(\frac{4}{x} + \frac{4}{2x} = 1\right) 2x$$

$$8 + 4 = 2x$$

$$12 = 2x$$

$$6 = x$$

Time for Nick working alone 6 hours

Time for Mike working alone 12 hours

33. A parking lot is 50 feet longer than it is wide. Determine the dimensions of the parking lot if the area of the parking lot is 30,000 square feet.

$$w(w+50) = 30{,}000$$

$$w^2 + 50w = 30{,}000$$

$$w^2 + 50w - 30{,}000 = 0$$

$$(w+200)(w-150) = 0$$

$w = -200$ or $w = 150$

width = 150 ft

length = 200 ft

Bonus (3pts): If $xy = 28$ and $x + y = 7$, find the sum of the reciprocals of x and y.

$$\frac{1}{x} + \frac{1}{y} = \frac{y}{xy} + \frac{x}{xy}$$

$$= \frac{y+x}{xy} = \frac{7}{28} = \boxed{\frac{1}{4}}$$

Have a wonderful summer!