Elementary Algebra
SAMPLE Final Examination

NAME: Answer Key
Fall 2016

You will have 2 hours to complete this exam. You may use a calculator but must show all algebraic work in the space provided to receive full credit. Read all directions carefully, simplify all answers fully, and clearly indicate your answer. Good Luck!

Solve each equation. Show all algebraic work for full credit. *(3 points each)*

1) $30 = 22 - x$

$-22 \quad -22$

$8 = -x$

$\frac{8}{-1} = \frac{-x}{-1}$

$-8 = x$

1) $x = -8$

2) $\frac{3}{4}x - 6 = 12$

$+6 \quad +6$

$\frac{4}{3} \cdot \frac{3}{4}x = 18 \cdot \frac{4}{3}$

$x = 24$

2) $x = 24$

3) $2y + 5 = -6y + 29$

$+6y \quad +6y$

$8y + 5 = 29$

$-5 \quad -5$

$\frac{8y}{8} = \frac{24}{8}$

$y = 3$

3) $y = 3$

Solve each equation. Show all algebraic work for full credit. *(3 points each)*

4) $5(4y-2)=70$ Distribute

4) $y=4$

$20y-10=70$

$+10 \quad +10$

$\frac{20y}{20}=\frac{80}{20}$

$y=4$

5) $\frac{1}{4}x+\frac{1}{3}=\frac{3}{4}x$ LCM = 12

5) $x=\frac{2}{3}$

$\frac{12}{1}\cdot\frac{1}{4}x+\frac{12}{1}\cdot\frac{1}{3}=\frac{12}{1}\cdot\frac{3}{4}x$

Clearing fractions

$3x+4=9x$

$-3x \quad -3x$

$\frac{4}{6}=\frac{6x}{6}$

$\frac{2}{3}=x$

6) $12x-(2x-6)=-20-3x$ → Distribute

6) $x=-2$

$12x-2x+6=-20-3x$ → Combine Like terms on each side separately.

$10x+6=-20-3x$

$+3x \quad +3x$

$13x+6=-20$

$-6 \quad -6$

$\frac{13x}{13}=\frac{-26}{13}$

$x=-2$

Solve the equation. Show all algebraic work for full credit. *(3 points)*

7) $4(y+1) = 5(y-5) - 2y$ → Distribute

7) $y = -29$

$4y + 4 = 5y - 25 - 2y$

$4y + 4 = 3y - 25$

$-3y \quad -3y$

$y + 4 = -25$

$-4 \quad -4$

$y = -29$

Solve each inequality and graph the solution on the number line provided. *(3 points each)*

8) $7x \leq 3x + 16$

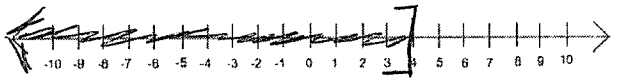
$-3x \quad -3x$

$\frac{4x}{4} \leq \frac{16}{4}$

$x \leq 4$

* Remember to reverse the direction of the inequality when you multiply or divide by a negative number.

8) $x \leq 4$



9) $100(1.25 + 3.75x + 7.5x > -1.5x + 26.75)100$

9) $x > 2$

Clear Decimals

$125 + 375x + 750x > -150x + 2675$

$125 + 1125x > -150x + 2675$

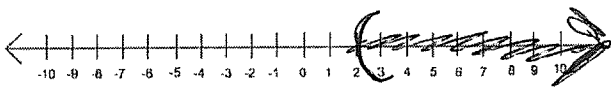
$+150x \quad +150x$

$125 + 1275x > 2675$

$-125 \quad -125$

$\frac{1275x}{1275} > \frac{2550}{1275}$

$x > 2$

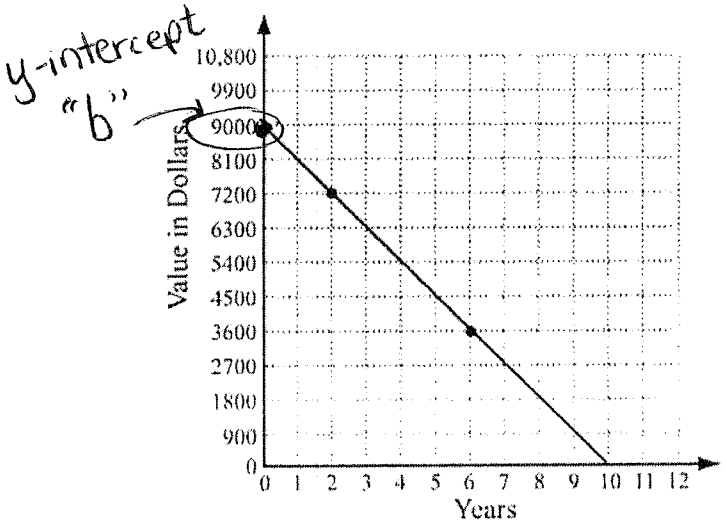


10) Write the equation of a line that has a slope of $-\frac{1}{2}$ and intersects the y-axis at (0, 11). *(2 points)*

$y = mx + b$

$y = -\frac{1}{2}x + 11$

11) Write the equation of the line for the graph shown. *(2 points)*



$y = mx + b$

$y = -900x + 9000$

12) Find the slope of each line. Then state whether the two lines are parallel, perpendicular or neither. Show your work and justify your answer for full credit. *(3 points)*

$y = \left(-\frac{1}{4}\right)x + 1$

Slope of first line $-\frac{1}{4}$

$-4x + y = 8$

$+4x \quad +4x$

$y = (4)x + 8$

Slope of second line 4

Answer with reason: The lines are perpendicular because the slopes are opposite reciprocals.

13) Given the line $5x + 3y = 15$, find the following. *(1 point each)*

x	y
0	5
3	0

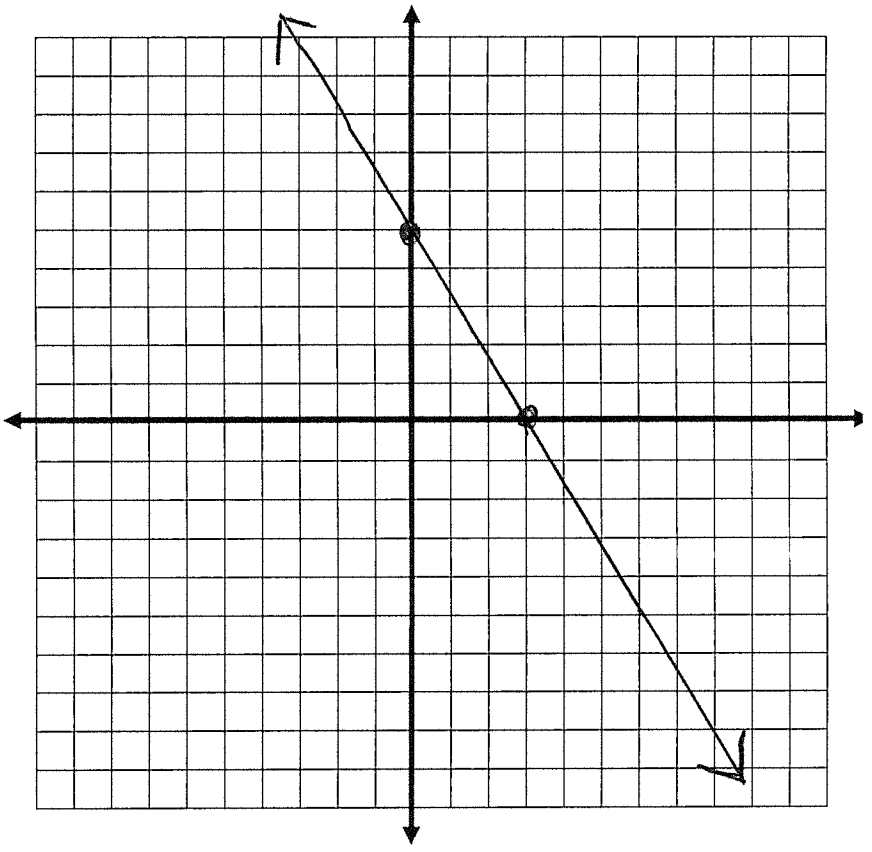
OR

$5x + 3y = 15$

$-5x \quad -5x$

$\frac{3y}{3} = \frac{-5x}{3} + \frac{15}{3}$

$y = -\frac{5}{3}x + 5$



a) x-intercept: $(3, 0)$

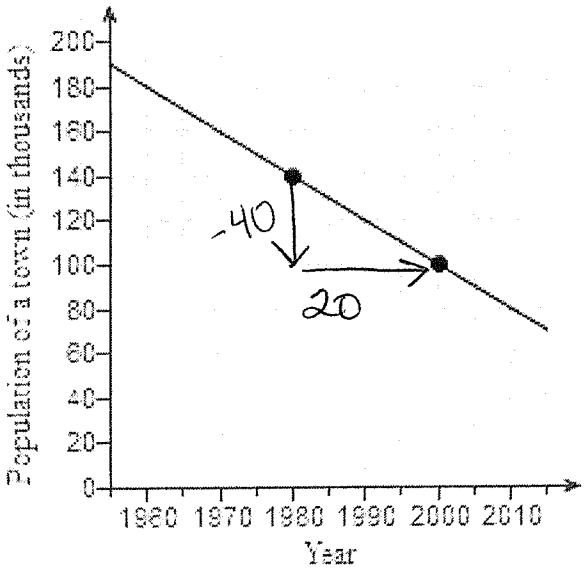
b) y-intercept: $(0, 5)$

c) slope: $-\frac{5}{3}$

d) Graph.

14) Use the graph to find the rate that the population was changing over time. Include the proper units in your answer. *(1 point)*

↳ $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$



$\frac{-40}{20} = -2$ thousand per year

14) $-\$2{,}000$ per year

15) The cost to enter a state park is \$6.00 per car plus \$2.00 for each person in the car.

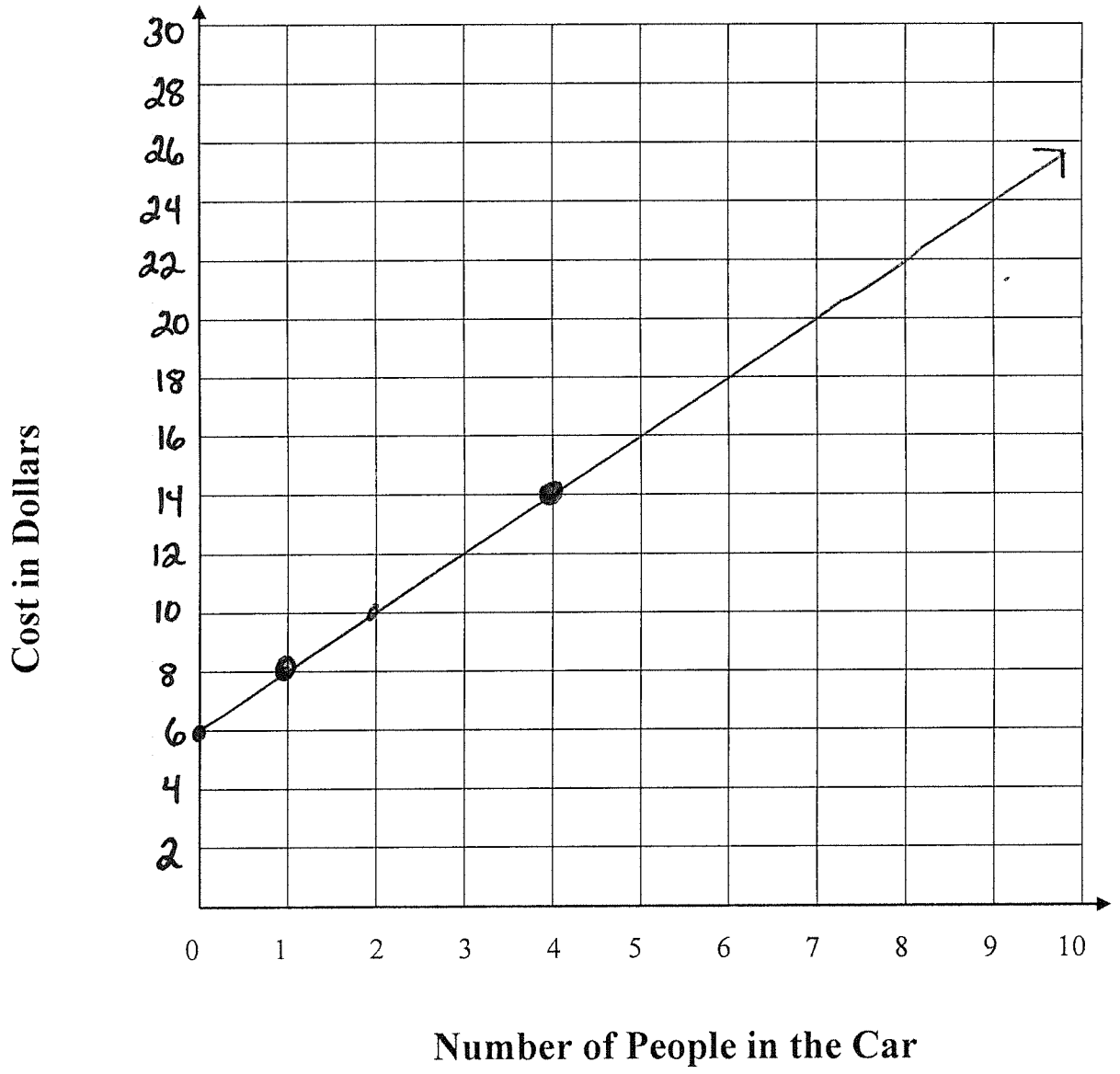
a) What would the cost be if there is 1 person in the car? \$8 *(2 point)* \$6 + \$2(1)

b) What would the cost be if there are 4 people in the car? \$14 *(2 point)* \$6 + \$2(4)

c) Write an equation that models this situation.
Let $y =$ *the cost* and let $x =$ *the number of people in the car.*

Equation: $y = 6 + 2x$ *(2 points)*
or $y = 2x + 6$

d) Graph. *(2 points)*



$x_1\ y_1 \quad x_2\ y_2$

16) Write the equation of the line that contains the points (-10, 8) and (5, 11). *(3 points)*

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-8}{5-(-10)} = \frac{3}{15} = \frac{1}{5} = m$$

$y = mx + b$

$11 = \frac{1}{5}(5) + b$

$11 = 1 + b$

$-1 \quad -1$

$10 = b$

OR

$y - 11 = \frac{1}{5}(x - 5)$

$y - 11 = \frac{1}{5}x - 1$

$+11 \qquad +11$

$y = \frac{1}{5}x + 10$

$y = \frac{1}{5}x + 10$

17) If $f(x) = x^2 + 9x + 18$, then find $f(0)$ *and* $f(-2)$. *(1 point each)*

a) $f(0) = (0)^2 + 9(0) + 18$

17a) 18

b) $f(-2) = (-2)^2 + 9(-2) + 18$

$4 - 18 + 18$

4

17b) 4

18) Find the point of intersection of the two lines using the substitution method. (3 points)

$3x + 2y = 8$
$x = y + 11$ substitute.

$3(y+11) + 2y = 8$

$3y + 33 + 2y = 8$

$5y + 33 = 8$

$-33 \quad -33$

$\frac{5y}{5} = \frac{-25}{5}$

$y = -5$

$x = y + 11$

$x = (-5) + 11$

$x = 6$

(x, y)

$(6, -5)$

19) Find the point of intersection of the two lines using the elimination (addition) method. *(3 points)*

$-5(x + 3y = 19)$
$5x - 2y = 10$

$-5x - 15y = -95$

$+ \quad 5x - 2y = 10$

$\frac{-17y}{-17} = \frac{-85}{-17}$

$y = 5$

$x + 3(5) = 19$

$x + 15 = 19$

$-15 \quad -15$

$x = 4$

(x, y)

$(4, 5)$

20) Express 0.000042 in scientific notation. *(1 point)*

21) 4.2×10^{-5}

21) Write 5.25×10^{7} in standard form (decimal notation). *(1 point)*

22) 52,500,000

22) Multiply. Write your answer in scientific notation. *(2 points)*

$(4 \times 10^{-8})(2.3 \times 10^{12})$

23) 9.2×10^{4}

Simplify each expression. Write the result using positive exponents. Please circle your final answer. *(2 point each)*

23) $x \cdot x^{14} \cdot x^{-3}$

x^{12}

25) $(y^{8})^{3}$

y^{24}

26) $(2x^{5}y^{-2})^{4}$

$16x^{20}y^{-8}$

$\frac{16x^{20}}{y^{8}}$

27) $\frac{9a^{10}b^{5}}{15a^{3}b^{9}}$

$\frac{3a^{7}}{5b^{4}}$

Perform the indicated operations. Simplify answers fully. (2 points each)

28) $(12x^3 - 6x - 1) + (3x^3 + x + 7)$

$15x^3 - 5x + 6$

29) $(11x^2 - 4x + 2) - (3x^2 + 2x - 8)$ Distribute the negative first.

$11x^2 - 4x + 2 - 3x^2 - 2x + 8$

$8x^2 - 6x + 10$

30) $7x(2x^2 + 6x - 1)$

$14x^3 + 42x^2 - 7x$

31) $(2x + 9)^2$

rewrite! $(2x+9)(2x+9)$

Then FOIL $4x^2 + 18x + 18x + 81$

Combine Like Terms $4x^2 + 36x + 81$

Perform the indicated operations. Simplify answers fully. (2 points each)

32) $(4p+5)(4p-5)$ F.O.I.L.

$16p^2 - 20p + 20p - 25$

$16p^2 - 25$

33) $(4x+3y)(x+2y)$ F.O.I.L.

$4x^2 + 8xy + 3xy + 6y^2$

$4x^2 + 11xy + 6y^2$

34) $\frac{27x^4 - 9x^3 + 15x^2}{-3x^2}$

$\frac{27x^4}{-3x^2} + \frac{-9x^3}{-3x^2} + \frac{15x^2}{-3x^2}$

$-9x^2 + 3x - 5$

Applications. Show your algebraic work for each problem. Include the proper units. Circle your final answer.

35) A walkway rises 6 feet vertically over a horizontal distance of 74 feet. What is the grade of the walkway as a percent? Round to the nearest tenth of a percent. *(2 points)*

run

$$\text{grade} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{6}{74} = 0.08108$$

percent

8.108%

round

8.1%

36) Young's Rule of Medicine states that for a child who is a years old, the dosage c of medicine they should receive when the adult dosage is d is given by:

$$c = \frac{ad}{a+12}$$

d c

If an adult dosage of an antibiotic is 150 milligrams, then what dosage of medicine should a 6-year-old child get? *(2 points)*

a

$$c = \frac{(6)(150)}{6+12} = \frac{900}{18} = 50\text{ mg}$$

Choose 4 out of the following 5 word problems to complete. Please put a large X through the problem that you do not want graded. Show all work for each problem.** ***(3 points each)

37) You paid \$16.26 for a meal including a 20% tip. How much was the cost before the tip? Round your answer to two decimal places.

x = cost of meal before tip

$x + .2x = 16.26$

$\frac{1.2x}{1.2} = \frac{16.26}{1.2}$

$x = \$13.55$

38) The equation $C = 2d + 4.5$ can be used to determine the cost of a taxi ride C, in dollars, given the number of miles d that the taxi travels. Determine the distance a taxi drove for a ride that costs \$54.50.

C d

$\$54.50 = 2d + 4.5$

$-4.5 \quad -4.5$

$\frac{50}{2} = \frac{2d}{2}$

$d = 25$ miles

39) Henry's financial aid stipulates that his tuition not exceed \$2,500. If his local community college charges a \$160 registration fee plus \$1,100 per course, what is the greatest number of courses for which ~~Karen~~ Henry can register? Show your work.

x = number of courses

$160 + 1100x \leq 2500$

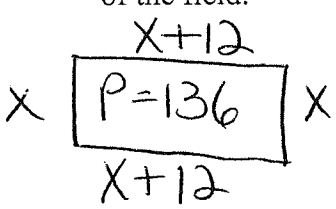
$-160 \quad -160$

$\frac{1100x}{1100} \leq \frac{2340}{1100}$

$x \leq 2.12\overline{7}$

The greatest number of courses he can register for is 2.

40) The perimeter of a field is 136 feet. The length is 12 feet longer than the width. Find the dimensions of the field.



$X + X+12 + X + X+12 = 136$

$4X + 24 = 136$

$-24 \quad -24$

$\frac{4X}{4} = \frac{112}{4}$

$X = 28$ feet

$X+12$

$28+12$

$= 40$ feet

The field is 28 feet by 40 feet.

41) A trip to New York City costs $40 for children and $60 for adults. A total of 55 people went on the trip and $2,840 was collected altogether. How many child tickets were sold? How many adult tickets were sold?

x = number of adult tickets sold $60

y = number of child tickets sold $40

55 people total

$2,840 collected

$-60(x + y = 55) \; -60$

$60x + 40y = 2840$

$-60x - 60y = -3300$

$+ \; 60x + 40y = 2840$

$-20y = -460$

$-20 \quad -20$

$y = 23$ children

$x + 23 = 55$

$-23 \quad -23$

$x = 32$ adults

BONUS: ***(2 points)***

Find two integers that have a sum of 13 and a product of -30

Bonus: 15 and -2