(SAMPLE FINAL EXAM for SPRING 2017)

Intermediate Algebra
~~Final Exam Fall 2016~~

NAME Answer Key

You will have 2 hours to complete this exam. You may use a calculator (TI-84 or lower, no cell phones) but must show all algebraic work in the space provided to receive full credit. Read all directions carefully, simplify all answers fully, and clearly indicate your answer. Good Luck!

Factor Completely. If the polynomial is prime, say so. (2pts each)

1. $5x^2 - 19x - 4$

$(5x+1)(x-4)$

2. $25 - 16x^2$

$(5)^2 - (4x)^2 = (5+4x)(5-4x)$

Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$ and, where appropriate, rationalize all denominators. (3 pts each)

3. $\frac{4x-8}{x+2} \div \frac{8x-16}{x^2-4}$

$\frac{4x-8}{x+2} \cdot \frac{x^2-4}{8x-16} = \frac{4(x-2)}{x+2} \cdot \frac{(x+2)(x-2)}{8(x-2)} = \frac{x-2}{2}$

4. $\frac{x^2}{x-3} + \frac{9}{3-x}$

$\frac{x^2}{x-3} - \frac{9}{x-3} = \frac{x^2-9}{x-3} = \frac{(x+3)(x-3)}{x-3} = x+3$

10

Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$ and, where appropriate, rationalize all denominators. (3 pts each)

5. $\dfrac{2x}{x^2-y^2} - \dfrac{1}{x-y}$

$$\frac{2x}{(x+y)(x-y)} - \frac{1}{x-y} = \frac{2x-1(x+y)}{(x+y)(x-y)} = \frac{2x-x-y}{(x+y)(x-y)}$$

$$= \frac{x-y}{(x+y)(x-y)}$$

$$= \frac{1}{x+y}$$

6. $\sqrt[3]{54x^5y^7}$

$$\sqrt[3]{3^3(2)\,x^3 \cdot x^2\,(y^2)^3 \cdot y} = 3xy^2\sqrt[3]{2x^2y}$$

7. $\sqrt{\dfrac{2}{5}}$

$$= \frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2}\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}} = \frac{\sqrt{10}}{5}$$

8. $6\sqrt{18} - 9\sqrt{8}$

$$6\sqrt{3^2(2)} - 9\sqrt{2^2(2)}$$

$$18\sqrt{2} - 18\sqrt{2} = 0$$

(12

Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$ and, where appropriate, rationalize all denominators. (3 pts each)

9. $(6 - \sqrt{7})(5 - \sqrt{7})$

$30 - 6\sqrt{7} - 5\sqrt{7} + 7 = 37 - 11\sqrt{7}$

10. $(9 - 7\sqrt{3}) - (6 - 5\sqrt{3})$

$9 - 7\sqrt{3} - 6 + 5\sqrt{3} = 3 - 2\sqrt{3}$

11. $(3 - i)(5 - 6i)$

$15 - 18i - 5i + 6i^2$

$15 - 23i - 6 = 9 - 23i$

12. $\frac{3}{5+3i} = \frac{3(5-3i)}{(5+3i)(5-3i)} = \frac{15-9i}{(5)^2-(3i)^2} = \frac{15-9i}{25-9i^2}$

$= \frac{15-9i}{25+9} = \frac{15-9i}{34}$

$= \frac{15}{34} - \frac{9}{34}i$

12

List any restrictions on the domain of each function below. (1pt each)

13. $f(x) = -5x^2 - 8x - 3$

No restriction since $f(x)$ is a polynomial

14. $f(x) = \frac{x+1}{x^2-x-12}$

$x^2 - x - 12 \neq 0$

$(x-4)(x+3) \neq 0$

$x - 4 \neq 0$; $x + 3 \neq 0$

$x \neq 4$; $x \neq -3$

Restrictions are -3 and 4.

15. $f(x) = \sqrt{2-x}$

Restriction: $2 - x < 0$

$2 < x$

16. **(2pts)** Given $f(x) = 2x^2 - 5x - 7$, find $f(-1)$

$f(-1) = 2(-1)^2 - 5(-1) - 7$

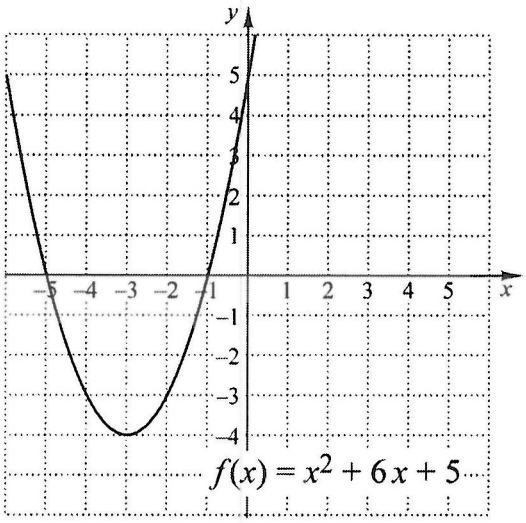
$= 2(1) + 5 - 7$

$= 2 + 5 - 7 = 0$

$f(-1) = 0$

5

17. Use the graph of the function below to determine the following. **(2pts each)**



a). Does the function have a maximum or a minimum value? And what is that value? Minimum ; Min. value = -4

b). What is the range of the function? $[-4, \infty)$

c). What are the zeros of the function? -5 and -1

18. For the quadratic function $f(x) = x^2 + 4x - 5$, find the following and graph. **(2pts each)**

Vertex (h, k)

a). Vertex $(-2, -9)$

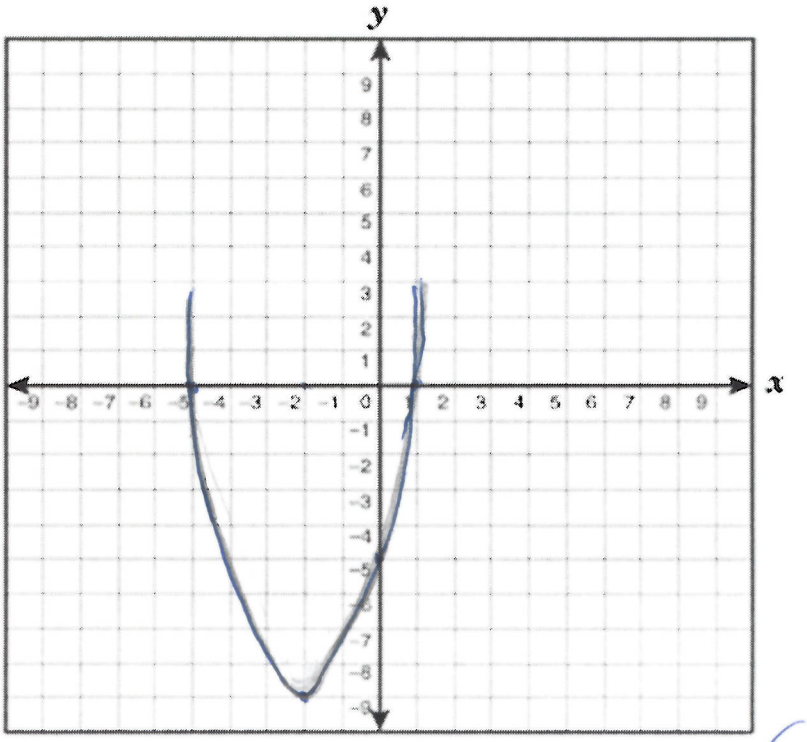
a) $h = \frac{-4}{2(1)} = -2$; $k = (-2)^2 + 4(-2) - 5$
$= 4 - 8 - 5 = -9$

b). $x^2 + 4x - 5 = 0$
$(x+5)(x-1) = 0$
$x = -5$; $x = 1$

b). x-intercept(s) -5 and 1
or
$(-5, 0)$ and $(1, 0)$

c). y-intercept -5 or $(0, -5)$

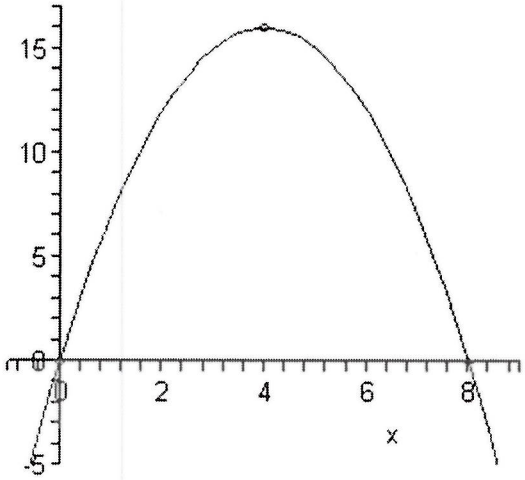
d). Graph to the right



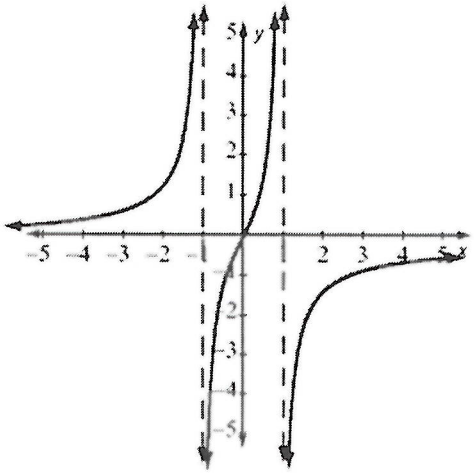
Match the graph to the type of function that best describes it. The same type may be used multiple times or not at all. (2pts each)

a). Linear b). Quadratic c). Exponential d). Radical e). Rational

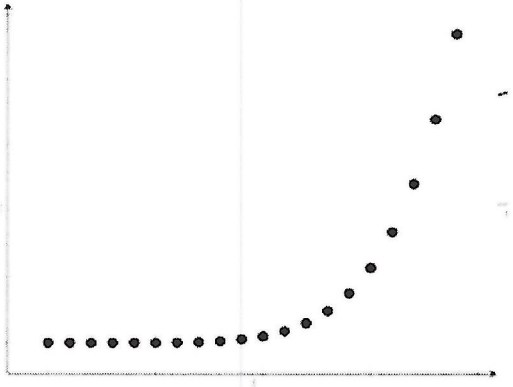
19. Quadratic (b)



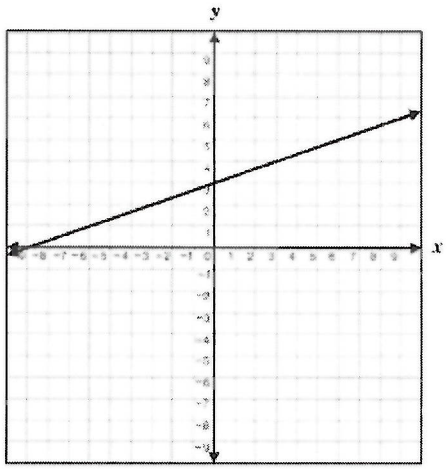
20. Rational (e)



21. Exponential (c)



22. Linear (a)



8

Solve each equation below. Simplify completely, do not round. (3 pts each)

23. $15x^2 - 14x = 8$

$15x^2 - 14x - 8 = 0$

$(5x+2)(3x-4) = 0$

$5x+2=0$; $3x-4=0$

$x = -\frac{2}{5}$; $x = \frac{4}{3}$

24. $x = 3 + \sqrt{5-x}$

$x - 3 = \sqrt{5-x}$

$(x-3)^2 = (\sqrt{5-x})^2$

$x^2 - 6x + 9 = 5 - x$

$x^2 - 6x + 9 - 5 + x = 0$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x - 4 = 0$; $x - 1 = 0$

$x = 4$; $x = 1$ (Extraneous)

only solution

25. $\frac{3}{x-3} + \frac{5}{x+2} = \frac{5x}{x^2-x-6}$

$\frac{3}{x-3} + \frac{5}{x+2} = \frac{5x}{(x-3)(x+2)}$; LCD $= (x-3)(x+2)$; Restriction: $x \neq 3$; $x \neq -2$

$3(x+2) + 5(x-3) = 5x$

$3x + 6 + 5x - 15 = 5x$

$3x - 9 = 0$

$3x = 9$

$x = 3$ (Extraneous)

no solution

9

Solve each equation below. Simplify completely, do not round. (3 pts each)

26. $\sqrt[5]{1-3x} = -1$

$$\left(\sqrt[5]{1-3x}\right) = (-1)^5$$

$$1 - 3x = -1$$

$$-3x = -2$$

$$x = 2/3$$

27. $x^2 - 10x + 29 = 0$

$$x = \frac{10 \pm \sqrt{(10)^2 - 4(1)(29)}}{2}$$

$$= \frac{10 \pm \sqrt{100 - 116}}{2}$$

$$= \frac{10 \pm \sqrt{-16}}{2}$$

$$= \frac{10 \pm 4i}{2}$$

$$x = 5 \pm 2i$$

6

Application Problems. For all problems where an equation is not given, you need to define your variable(s), set up an algebraic equation or equations, solve algebraically, and answer the question with the proper units. If an equation is given, be sure to answer the question completely and with proper units. (4 pts each)

28. The height of waves in a storm depends on the speed of the wind. Suppose H(x) = $2x^2 - 38x - 35$ gives the maximum wave height H in feet for a wind speed x in knots (nautical miles per hour). For what wind speed would the maximum wave height be 5 feet?

$H(x) = 5$

$2x^2 - 38x - 35 = 5$

$2x^2 - 38x - 40 = 0$

$2(x^2 - 19x - 20) = 0$

$2(x-20)(x+1) = 0$

$x - 20 = 0$ or $x + 1 = 0$

$x = 20$ $x \neq -1$

A wind speed of 20 knots would yield a maximum wave height of 5 feet.

29. Suppose that the number of pounds, p(x), of meat per day recommended for a lion cub that is x days old is given by $p(x) = -0.2x^2 + 1.6x + 10.8$

a). At what age is the cub's consumption of meat maximum? (2 pts)

$x = \frac{-b}{2a}$ $\begin{cases} a = -0.2 \\ b = 1.6 \end{cases}$

$x = \frac{-1.6}{2(-0.2)} = \frac{-1.6}{-0.4} = 4$

At 4 days old, the cub's meat consumption is maximum

b). How many pounds of meat does the cub consume at its age of maximum meat consumption? (2 pts)

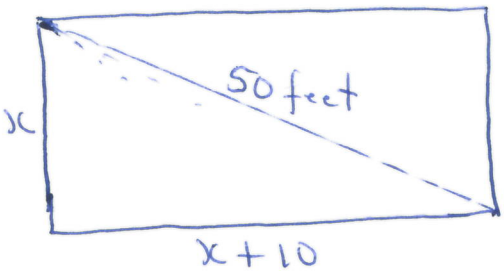
# of pounds of meat = $p(4) = -0.2(4)^2 + 1.6(4) + 10.8$

$= -3.2 + 6.4 + 10.8$

$= 14$

14 pounds of meat

30. The diagonal of a rectangular gate is 50 feet. The horizontal distance that it spans is 10 feet longer than its height. Find the length and height of the gate.



$x^2 + (x+10)^2 = (50)^2$

$x^2 + x^2 + 20x + 100 = 2500$

$2x^2 + 20x - 2400 = 0$

$2(x^2 + 10x - 1200) = 0$

$2(x+40)(x-30) = 0$

$x+40 = 0$ or $x-30 = 0$

$x = -40$ $\quad x = 30$

Length of gate is: 40 feet

Height of gate is: 30 feet

31. Hans' boat travels 8 mph faster than Julie's. Hans travels 69 miles in the same time that Julie travels 45 miles. Find the speed of each person's boat.

Let r be the speed of Julie's boat. Let $(r+8)$ be the speed of Hans' boat. { Let t be time in hours.

$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$.

Julie's time → $\frac{69}{r+8} = \frac{45}{r}$ ← Hans' time

$69r = 45(r+8)$

$69r = 45r + 360$

$24r = 360$

$r = 15$

Speed of Hans' boat is: 23 mph

Speed of Julie's boat is: 15 mph

8

32. Amy can paint her four-bedroom condominium three times as fast as her husband, Paul. If they work together, it takes them 12 hours to paint the condominium. How long would it take Paul to paint the entire condominium alone?

x = Amy's time to paint the Condominium alone.

$3x$ = Paul's time .....

$$\frac{1}{x} + \frac{1}{3x} = \frac{1}{12} \quad ; \quad LCD = 12x$$

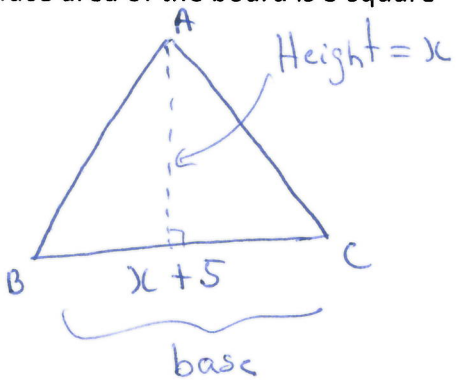
Clear Fractions: $\frac{12x}{x} + \frac{12x}{3x} = \frac{12x}{12}$

$$12 + 4 = x$$

$$x = 16$$

It would take Paul 48 hours to paint the Condominium alone.

33. The base of a triangular board is 5 feet longer than its height. If the surface area of the board is 3 square feet, then find the base and height of the board.



$$(\text{Base})(\text{Height}) = 2(\text{Area})$$

$$(x+5)(x) = 2(3)$$

$$x^2 + 5x = 6$$

$$x^2 + 5x - 6 = 0$$

$$(x+6)(x-1) = 0$$

$$x + 6 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x \neq -6 \qquad x = 1$$

Base of board is: 6 feet

Height of board is: 1 foot

Bonus (3pts)

There is a special 5-digit number that has the following features: If we put the numeral 1 at the beginning, we get a number three times smaller than if we put the numeral 1 at the end. What is that special 5-digit number?

Let x be such 5-digit number

a). The expression $(100,000 + x)$ adds the numeral 1 at the beginning of x.

b). The expression $(10x + 1)$ adds the numeral 1 at the end of x.

So,

$$3(100,000 + x) = 10x + 1$$

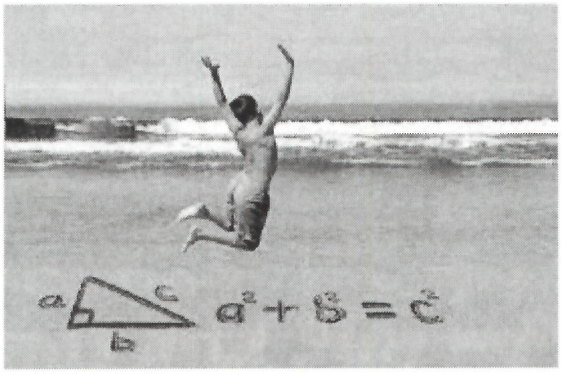
$$300000 + 3x = 10x + 1$$

$$299999 = 7x$$

$$x = \frac{299999}{7}$$

$$x = 42857$$

Merry Christmas and Happy New Year!



3