	Linear Functions	Exponential Functions	Quadratic Functions

Tables

Linear Functions

Constant 1st differences in output (when inputs increase by a constant)

Δ input	t = time spent on repair work (hours)	C(t) = Total cost of repair work ($)	Δ output 1st Diff.
	0	50	
+ 1 hr.	1	95	+ $45
+ 1 hr.	2	140	+ $45
+ 1 hr.	3	185	+ $45

Exponential Functions

Constant multipliers between outputs (when inputs increase by a constant)

	d # days	**N(d)** # cells after d days	
	0	45,000	
+1	1	30,000	× 2/3
+1	2	20,000	× 2/3
+1	3	13,333	× 2/3

Quadratic Functions

Constant 2nd differences in output (when inputs increase by a constant)

Input Diff	s (feet)	A(s) (sq. ft.)	1st Dif	2nd Diff
	0	0		
+ 1	1	1	+ 1	
+ 1	2	4	+ 3	+ 2
+ 1	3	9	+ 5	+ 2
+ 1	4	16	+ 7	+ 2
+ 1	5	25	+ 9	+ 2

Equations

x = input
= Independent var.
y = f(x) = output
= Dependent var.

Linear: $y = f(x) = \mathbf{m}x + \mathbf{b}$

Exponential: $y = f(x) = \mathbf{a}(\mathbf{b})^x$
$y = f(x) = \mathbf{a}(1 \pm \mathbf{r})^x$

Quadratic: $y = f(x) = \mathbf{a}x^2 + \mathbf{b}x + \mathbf{c}$

Function Equations

Meaning of Constants

Linear Functions

b – initial (output) **value** when input=0
(0, b) is **'y'-intercept**

m – constant rate of change (CRC)
m – slope (steepnes & direction of graph)

$$\mathbf{m} = \frac{Change\ in\ Outputs}{Change\ in\ Inputs} = \frac{\Delta y}{\Delta x} = \frac{rise}{run}$$

$$\mathbf{m} = \frac{+45\ \$}{+1\ \text{hr.}} = 45\ \$/\text{hour}$$ (in table above)

Exponential Functions

a – initial (output) **value** when input=0
(0, a) is **'y'-intercept**

b – constant multiplier (or ratio)

$$\mathbf{b} = \frac{Output}{Previous\ Ouput}$$

b = ratio: output to previous output

r = **relative rate of change** (growth/decay rate)
b = (1 $\pm$ **r**) when **r** = relative rate of change

b = 2/3 , **r** = 1/3 (in table above)

Quadratic Functions

c – initial (output) **value** when input=0
(0, c) is **'y'-intercept**

The meanings of **a** and **b** depend on the situation and are more involved than we will address in this class.

In projectile motion functions,
a is half of the acceleration due to gravity and **b** is the initial vertical velocity. EX: $h(t) = -16t^2 + 30t + 5$

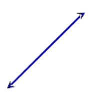
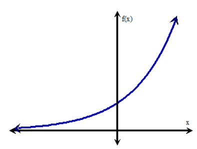
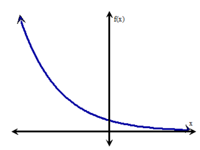
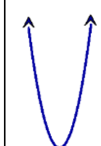
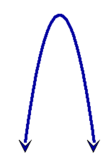
Equations

Restrictions on Constants

Linear: **m** – none , **b** – none

Exponential: **b** ≠ 0, 1, **b** > 0, **a** > 0 in real world

Quadratic: **a** ≠ 0, **b**, **c** none

	Linear Functions	Exponential Functions	Quadratic Functions
Rates Linear versus Exponential	**m** = **Constant Rate of Change** (CRC) *changes by a constant quantity* which must include units. **<u>EX:</u>** The population of a town was 10,000 in 2010 and grew by 200 people per year. **m** = CRC = +200 people per year The pop. changes by the same quantity – 200 people each year. P(t) = 10,000 + 200t	**r** = **Constant Relative Rate of Growth/Decay** is a *constant fractional* (or percent) *change relative to the previous* amount. The fractional change is not a quantity and therefore unitless. **<u>EX:</u>** The population of a town was 10,000 in 2010 and grew by 2% (or 1/50) each year. **r** = +2% = 1/50 of previous year's pop. per year The population changes by the same fraction but it's the same fraction of a different amount ea. yr., so the amount of change is different each yr. $P(t) = 10{,}000(1 + 0.02)^t = 10{,}000(1.02)^t$	**N/A**
Graphs Shape & Direction	**Straight Lines** - including horizontal lines (constant)  Increasing (linear growth) **m** > 0 (pos.)  Decreasing (linear decay) **m** < 0 (neg.) Constant (output always same) **m** = 0 (horizontal line) ***Non-function*** (EQ: x = #) m undefined (vertical line)	**Asymptotic**, Always above x-axis (y > 0) *Asymptote* – line the graph gets closer and closer to without touching or crossing  Increasing (exponential growth) **b** > 1, **b** = (1 + **r**) x-axis is an asymptote  Decreasing (exponential decay) 0 < **b** < 1, **b** = (1 – **r**)	**Parabolas** – special U-shapes - have focus point (not on graph) that make them useful for satellite dishes, solar collectors, and headlights  Opens UP **a** > 0 (pos.) Vertex is a MIN  Opens DOWN **a** < 0 (neg.) Vertex is a MAX

Intercepts — y-intercept

y-intercept

x = 0

(0, __)

-initial (output) value

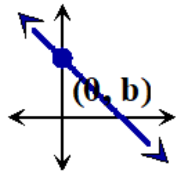
-functions have ONE y-intercept at most

Linear Functions

y-intercept: f(0) = m(0) + b

f(0) = 0 + b = b

(0, b) - b is the initial value

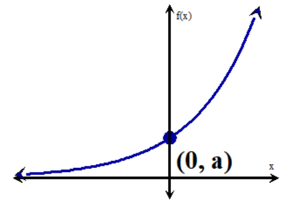


Exponential Functions

y-intercept: $f(0) = a(b)^0$

f(0) = a·1 = a

(0, a) - a is the initial value

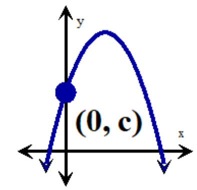


Quadratic Functions

y-intercept: $f(0) = a(0)^2 + b(0) + c$

f(0) = 0 + 0 + c = c

(0, c) - c is the initial value



Intercepts — x-intercept(s)

x-intercept(s)

y = 0

(__, 0)

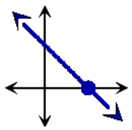
-functions can have multiple x-int. or none at all

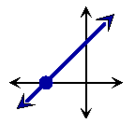
Linear Functions

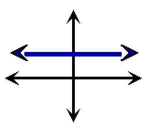
x-intercept(s): 0 = mx + b

solve to find the x value

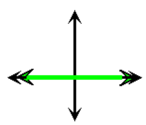
-**Non-Constant Linears** have **1 x-int.**







-**Constant Linears** (y = #) have **NO x-int.**



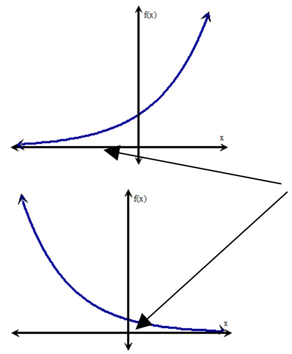
Constant zero function y=0 has **infinitely many**

Exponential Functions

x-intercept(s): $0 = a(b)^x$

solve to find the x value – *impossible*

-**Exponential** Growth/Decay functions **DO NOT HAVE x-intercepts**.



x-axis is an asymptote

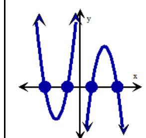
Quadratic Functions

x-intercept(s): $0 = ax^2 + bx + c$

solve to find the x value(s)

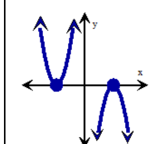
-Use **Quadratic Formula** to solve

-Parabolas can have:



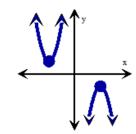
2 x-intercepts

$$x = \frac{-b \pm \sqrt{\textbf{positive \#}}}{2a}$$



1 x-intercept – also vertex

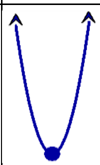
$$x = \frac{-b \pm \sqrt{\mathbf{0}}}{2a}$$



0 x-intercepts

$$x = \frac{-b \pm \sqrt{\textbf{negative \#}}}{2a}$$

Vertex — Quadratics ONLY

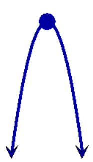


Vertex a MINIMUM point

Lowest point on parabola

Least output value

OPENS UP , a > 0



Vertex a MAXIMUM point

Highest point on parabola

Greatest output value

OPENS DOWN , a < 0

To find vertex: (x, y) **(Quadratic Graphs – Parabolas)**

1st x-coordinate: $\mathbf{x} = \frac{-b}{2a}$

2nd y-coordinate: 'plug in' the x-value

- *evaluate* function equation at x value

$$\mathbf{y} = a\left(\frac{-b}{2a}\right)^2 + b\left(\frac{-b}{2a}\right) + c$$

Solving Equations

Linear Equations

Variable highest power 1 (1st degree)

$$5y + 46 - 3y = 10 + 14y$$

-Simplify each side - Combine like terms.

5y + 46 – **3y** = 10 + 14y

2y + 46 = 10 + 14y

-Move all variable terms to one side and
-Move all constant terms to the other side.

$$\begin{array}{rcl} 2y + 46 & = & 10 + 14y \\ \underline{-2y\ -10} & \underline{=} & \underline{-10\ -2y} \\ 36 & = & 12y \end{array}$$

-Multiply/Divide B.S. by coefficient of variable to solve for final value of variable.

$$\frac{36}{12} = \frac{12y}{12}$$

$$3 = 1y = y$$

<u>Check:</u>

$$5(3) + 46 - 3(3) = 10 + 14(3)$$
$$15 + 46 - 9 = 10 + 42$$
$$15 + 46 - 9 = 10 + 42$$
$$52 = 52$$

Exponential Equations

Variable is IN the exponent.

$$900 = 60(1.2)^t$$

-Divide B.S. to Isolate the Exponential Term (base with its variable exponent).

$$\frac{900}{60} = \frac{60(1.2)^t}{60}$$

$$15 = 1(1.2)^t$$
$$15 = (1.2)^t$$

-Use logarithms to solve for the variable exponent.

$$15 = (1.2)^t$$

$$t = \log_{1.2}(15) = \frac{\log(15)}{\log(1.2)} = \frac{\ln(15)}{\ln(1.2)}$$

$$t \approx 14.85$$

<u>Check:</u>

$$900 = 60(1.2)^{14.85}$$
$$900 \approx 60(14.99)$$
$$900 \approx 899.4$$

Quadratic Equations

Variable highest power 2 (2nd degree)

$$-6x^2 + 9x + 8x^2 = 3x^2 + 5x - 12$$

-Simplify each side - Combine like terms.

$$\mathbf{-6x^2} + 9x \mathbf{+ 8x^2} = 3x^2 + 5x - 12$$

$$9x \mathbf{+ 2x^2} = 3x^2 + 5x - 12$$

-Move ALL terms to one side,
In other words, **set the equation = 0**

$$\begin{array}{rcl} 9x + 2x^2 & = & 3x^2 + 5x - 12 \\ \underline{-9x - 2x^2} & & \underline{-2x^2 \quad -9x} \\ \mathbf{0} & = & x^2 - 4x - 12 \end{array}$$

Now find a, b, c for the quadratic in standard form (=0) and use the **Quadratic Formula** to solve.

a = 1, b = –4, c = –12

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2(1)}$$
$$= \frac{4 \pm \sqrt{16+48}}{2} = \frac{4 \pm \sqrt{64}}{2} = \frac{4 \pm 8}{2}$$

$$x = \frac{4+8}{2} = 6 \quad \textit{or} \quad x = \frac{4-8}{2} = -2$$

<u>Check:</u>

$$-6(6)^2 + 9(6) + 8(6)^2 = 3(6)^2 + 5(6) - 12$$
$$-216 + 54 + 288 = 108 + 30 - 12$$
$$126 = 126$$

$$-6(-2)^2 + 9(-2) + 8(-2)^2 = 3(-2)^2 + 5(-2) - 12$$
$$-24 - 18 + 32 = 12 - 10 - 12$$
$$-10 = -10$$