### Linear Functions

**Tables**
- **Constant 1st differences in output (when inputs increase by a constant)**
  - | t (hours) | C(t) (Total cost of repair work ($)) | Δ output 1st Diff. |
  - | 0 | 50 | + $45 |
  - | 1 | 95 | + $45 |
  - | 2 | 140 | + $45 |
  - | 3 | 185 |

**Equations**
- \( y = f(x) = mx + b \)

**Function Equations**
- **b** – initial (output) value when input=0 (0, b) is ‘y’-intercept
- **m** – constant rate of change (CRC)
- **m** – slope (steepness & direction of graph)
- \( m = \frac{\text{Change in Outputs}}{\text{Change in Inputs}} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} \)
- \( m = \frac{+45}{+1 \text{ hr.}} = 45 \text{ $/hr. (in table above)} \)

**Restrictions on Constants**
- **m** – none , **b** – none

### Exponential Functions

**Tables**
- **Constant multipliers between outputs (when inputs increase by a constant)**
  - | d # days | N(d) # cells after d days |
  - | 0 | 45,000 \times \frac{2}{3} |
  - | 1 | 30,000 \times \frac{2}{3} |
  - | 2 | 20,000 \times \frac{2}{3} |
  - | 3 | 13,333 \times \frac{2}{3} |

**Equations**
- \( y = f(x) = a(b)^x \)
- \( y = f(x) = a(1+\textit{r})^x \)

**Function Equations**
- **a** – initial (output) value when input=0 (0, a) is ‘y’-intercept
- **b** – constant multiplier (or ratio)
- \( b = \frac{\text{Output}}{\text{Previous Output}} \)
- **b** = ratio: output to previous output
- \( r = \text{relative rate of change} \) (growth/decay rate)
- \( b = (1 \pm r) \) when \( r = \text{relative rate of change} \)
- \( b = \frac{2}{3} , \quad r = \frac{1}{3} \) (in table above)

**Restrictions on Constants**
- **a** ≠ 0, **b > 0, a > 0 in real world

### Quadratic Functions

**Tables**
- **Constant 2nd differences in output (when inputs increase by a constant)**
  - | S (feet) | A(s) (sq ft) |
  - | 0 | 0 |
  - | 1 | 1 |
  - | 2 | 4 |
  - | 3 | 9 |
  - | 4 | 16 |
  - | 5 | 25 |

**Equations**
- \( y = f(x) = ax^2 + bx + c \)

**Function Equations**
- **c** – initial (output) value when input=0 (0, c) is ‘y’-intercept

The meanings of **a** and **b** depend on the situation and are more involved than we will address in this class.

In projectile motion functions, **a** is half of the acceleration due to gravity and **b** is the initial vertical velocity. EX: \( h(t) = -16t^2 + 30t + 5 \)
<table>
<thead>
<tr>
<th>Linear Functions</th>
<th>Exponential Functions</th>
<th>Quadratic Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rates</strong></td>
<td><strong>Rates</strong></td>
<td><strong>N/A</strong></td>
</tr>
<tr>
<td><strong>Linear versus Exponential</strong></td>
<td><strong>Linear versus Exponential</strong></td>
<td></td>
</tr>
<tr>
<td><strong>m = Constant Rate of Change (CRC) changes by a constant quantity which must include units.</strong></td>
<td><strong>r = Constant Relative Rate of Growth/Decay is a constant fractional (or percent) change relative to the previous amount. The fractional change is not a quantity and therefore unitless.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>EX:</strong> The population of a town was 10,000 in 2010 and grew by 200 people per year. m = CRC = +200 people per year The pop. changes by the same quantity – 200 people each year. P(t) = 10,000 + 200t</td>
<td><strong>EX:</strong> The population of a town was 10,000 in 2010 and grew by 2% (or 1/50) each year. r = +2% = 1/50 of previous year’s pop. per year The population changes by the same fraction but it’s the same fraction of a different amount ea. yr., so the amount of change is different each yr. P(t) = 10,000(1 + 0.02)^t = 10,000(1.02)^t</td>
<td></td>
</tr>
<tr>
<td><strong>Graphs</strong></td>
<td><strong>Graphs</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Shape &amp; Direction</strong></td>
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<td></td>
</tr>
<tr>
<td><strong>Straight Lines</strong> - including horizontal lines (constant)</td>
<td><strong>Asymptotic</strong>, Always above x-axis (y &gt; 0) <strong>Asymptote</strong> – line the graph gets closer and closer to without touching or crossing</td>
<td><strong>Parabolas</strong> – special U-shapes - have focus point (not on graph) that make them useful for satellite dishes, solar collectors, and headlights</td>
</tr>
<tr>
<td>Increasing (linear growth) m &gt; 0 (pos.)</td>
<td>Increasing (exponential growth) b &gt; 1, ( b = (1 + r) ) x-axis is an asymptote</td>
<td>Opens UP ( a &gt; 0 ) (pos.) Vertex is a MIN</td>
</tr>
<tr>
<td>Decreasing (linear decay) m &lt; 0 (neg.)</td>
<td>( b = (1 - r) )</td>
<td>Opens DOWN ( a &lt; 0 ) (neg.) Vertex is a MAX</td>
</tr>
<tr>
<td>Constant (output always same) m = 0 (horizontal line)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Non-function</strong> m undefined (vertical line)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Linear Functions

<table>
<thead>
<tr>
<th><strong>y-intercept</strong></th>
<th><strong>y-intercept:</strong> $f(0) = m(0) + b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = 0$</td>
<td>$f(0) = 0 + b = b$</td>
</tr>
<tr>
<td>$(0, b)$</td>
<td>$b$ is the initial value</td>
</tr>
</tbody>
</table>

- Functions can have ONE $y$-intercept at most.

<table>
<thead>
<tr>
<th><strong>x-intercept(s)</strong></th>
<th><strong>x-intercept(s):</strong> $0 = mx + b$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>solve to find the $x$ value</td>
</tr>
</tbody>
</table>

- Non-Constant Lines have 1 $x$-int.
- Constant Lines ($y = #)$ have NO $x$-int.

### Exponential Functions

<table>
<thead>
<tr>
<th><strong>y-intercept:</strong></th>
<th><strong>y-intercept:</strong> $f(0) = a(b)^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(0) = a \cdot 1 = a$</td>
<td></td>
</tr>
<tr>
<td>$(0, a)$</td>
<td>$a$ is the initial value</td>
</tr>
</tbody>
</table>

- Exponential Growth/Decay functions DO NOT HAVE $x$-intercepts.

### Quadratic Functions

<table>
<thead>
<tr>
<th><strong>y-intercept:</strong></th>
<th><strong>y-intercept:</strong> $f(0) = a(0)^2 + b(0) + c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(0) = 0 + 0 + c = c$</td>
<td></td>
</tr>
<tr>
<td>$(0, c)$</td>
<td>$c$ is the initial value</td>
</tr>
</tbody>
</table>

- Functions can have multiple $x$-int. or none at all.

### Vertex

- Quadratics ONLY

<table>
<thead>
<tr>
<th><strong>Vertex a MINIMUM point</strong></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Lowest point on parabola</td>
<td>Least output value</td>
</tr>
<tr>
<td>OPENS UP, $a &gt; 0$</td>
<td></td>
</tr>
</tbody>
</table>

- Vertex a MAXIMUM point

<table>
<thead>
<tr>
<th><strong>Vertex a MAXIMUM point</strong></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Highest point on parabola</td>
<td>Greatest output value</td>
</tr>
<tr>
<td>OPENS DOWN, $a &lt; 0$</td>
<td></td>
</tr>
</tbody>
</table>

### To find vertex: $(x, y)$

1. **1st x-coordinate:** $x = \frac{-b}{2a}$
2. **2nd y-coordinate:** ‘plug in’ the $x$-value

- **evaluate** function equation at $x$ value

\[ y = a \left( \frac{-b}{2a} \right)^2 + b \left( \frac{-b}{2a} \right) + c \]
### Solving Equations

#### Linear Equations

**Variable highest power 1 (1st degree)**

5y + 46 – 3y = 10 + 14y  
- Simplify each side - Combine like terms.

5y + 46 = 10 + 14y  
2y + 46 = 10 + 14y  
- Move all variable terms to one side and move all constant terms to the other side.

2y + 46 = 10 + 14y  
-2y - 10 = -10 - 2y  
36 = 12y  
- Multiply/Divide B.S. by coefficient of variable to solve for final value of variable.

36 = 12y  
12 12  
3 = 1y = y  

**Check:**  
5(3) + 46 – 3(3) = 10 + 14(3)  
15 + 46 = 10 + 42  
15 + 46 – 9 = 10 + 42  
52 = 52  

### Exponential Equations

**Variable is IN the exponent.**

900 = 60(1.2)^t  
- Divide B.S. to Isolate the Exponential Term (base with its variable exponent).

900 = 60(1.2)^t  
60 60  
15 = 1(1.2)^t  
15 = (1.2)^t  
- Use logarithms to solve for the variable exponent.

t = \log_{1.2}(15) = \frac{\log(15)}{\log(1.2)}  
\approx 14.85  

**Check:**  
900 = 60(1.2)^{14.85}  
900 = 60(14.99)  
900 = 899.4  

### Quadratic Equations

**Variable highest power 2 (2nd degree)**

-6x^2 + 9x + 8x^2 = 3x^2 + 5x – 12  
- Simplify each side - Combine like terms.

-6x^2 + 9x + 8x^2 = 3x^2 + 5x – 12  
9x + 2x^2 = 3x^2 + 5x – 12  
- Move ALL terms to one side, in other words, set the equation = 0.

0 = x^2 – 4x – 12  

Now find a, b, c for the quadratic in standard form (=0) and use the Quadratic Formula to solve.

a = 1, b = -4, c = -12

x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2(1)}  
= \frac{4 + \sqrt{16 + 48}}{2} = \frac{4 + \sqrt{64}}{2} = \frac{4 + 8}{2}  
= \frac{12}{2} = 6  
or  x = \frac{4 - 8}{2} = -2  

**Check:**  
-6(6)^2 + 9(6) + 8(6)^2 = 3(6)^2 + 5(6) – 12  
-216 + 54 + 288 = 108 + 30 – 12  
126 = 126  

-6(-2)^2 +9(-2) +8(-2)^2 = 3(-2)^2 +5(-2) – 12  
-24 – 18 + 32 = 12 – 10 – 12  
-10 = -10  

### Exponential Equations

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- Divide B.S. to Isolate the Exponential Term (base with its variable exponent).

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### Quadratic Equations

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