Intermediate Algebra

NAME Answer Key

~~Final Exam Spring 2018~~

Sample Final Exam Fall 2018

**You will have 2 hours to complete this exam. You may use a calculator (TI-84 or lower, no cell phones) but must show all algebraic work in the space provided to receive full credit. Read all directions carefully, simplify all answers fully, and clearly indicate your answer. Good Luck!**

**Factor Completely. If the polynomial is prime, say so. (2pts each)**

1. $6x^2 - 13x - 5$

$(3x+1)(2x-5)$

2. $1 - 100x^2$

$(1+10x)(1-10x)$

**Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$ and, where appropriate, rationalize all denominators. (3 pts each)**

3. $\frac{2x-6}{x+3} \div \frac{5x-15}{x^2-9}$

$\frac{2x-6}{x+3} \cdot \frac{x^2-9}{5x-15} = \frac{2(x-3)}{x+3} \cdot \frac{(x+3)(x-3)}{5(x-3)} = \frac{2(x-3)}{5}$

4. $\frac{x^2}{x-3} + \frac{9}{3-x}$

$\frac{x^2}{x-3} - \frac{9}{x-3} = \frac{x^2-9}{x-3} = \frac{(x+3)(x-3)}{x-3} = x+3$



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**Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$ and, where appropriate, rationalize all denominators. (3 pts each)**

5. $\frac{7}{x+2} - \frac{3}{x-4}$

$$\frac{7(x-4) - 3(x+2)}{(x+2)(x-4)} = \frac{7x - 28 - 3x - 6}{(x+2)(x-4)} = \frac{4x - 34}{(x+2)(x-4)}$$

6. $\frac{x+1}{2x-5} - \frac{x+1}{2x-5} = 0$

opposites

7. $\frac{4}{\sqrt{6}} = \frac{4\sqrt{6}}{\sqrt{6} \cdot \sqrt{6}} = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3}$

8. $3\sqrt[3]{40} - 2\sqrt[3]{135}$

$$3\sqrt[3]{2^3(5)} - 2\sqrt[3]{3^3(5)}$$

$$3(2)\sqrt[3]{5} - 2(3)\sqrt[3]{5}$$

$$6\sqrt[3]{5} - 6\sqrt[3]{5} = 0$$



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**Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$ and, where appropriate, rationalize all denominators. (3 pts each)**

9. $(4 + 2\sqrt{3})(7 - 5\sqrt{3})$

$28 - 20\sqrt{3} + 14\sqrt{3} - 10(3)$

$28 - 6\sqrt{3} - 30 = -2 - 6\sqrt{3}$

10. $\frac{x-1}{2} - \frac{x-1}{3}$

$\frac{3(x-1) - 2(x-1)}{6} = \frac{3x - 3 - 2x + 2}{6} = \frac{x-1}{6}$

11. $(7 + 2i)(3 - 8i)$

$21 - 56i + 6i - 16i^2$

$21 - 50i - 16(-1)$

$21 - 50i + 16 = 37 - 50i$

12. $\frac{2}{3-5i}$

$\frac{2(3+5i)}{(3-5i)(3+5i)} = \frac{2(3+5i)}{(3)^2 - (5i)^2} = \frac{2(3+5i)}{9 - 25i^2} = \frac{2(3+5i)}{9+25}$

$= \frac{2(3+5i)}{34} = \frac{3+5i}{17} = \frac{3}{17} + \frac{5}{17}i$



12

**State the domain of each function below. (1pt each)**

13. $f(x) = -5x^2 - 3x + 2$

Domain = All real numbers since $f(x)$ is a polynomial.

Domain = $(-\infty, \infty)$

14. $f(x) = \frac{x-6}{x-3}$

$x - 3 \neq 0$ ; $x \neq 3$

Domain = All real numbers except $x = 3$

Domain = $\{x \mid x \neq 3\}$

15. $f(x) = \sqrt{x-1}$

$x - 1 \geq 0$

$x \geq 1$

Domain = All real numbers greater than or equal to 1

$= \{x \mid x \geq 1\}$

16. **(2pts)** Given $f(x) = 3x^2 - 4x + 5$, find $f(-2)$

$f(-2) = 3(-2)^2 - 4(-2) + 5$

$= 3(4) + 8 + 5$

$= 12 + 8 + 5 = 25$



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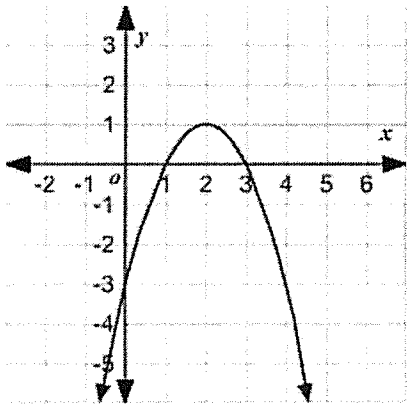
17. Use the graph of the function below to determine the following.




a). Does the function have a maximum or a minimum value? And what is that value? **(2pts)** Maximum Value = 1

b) What is the domain of the function? **(1 pt)** $(-\infty, \infty)$. All real #s.

c). What is the range of the function? **(1 pt)** $(-\infty, 1]$

d). What are the zeros of the function? **(2 pts)** 1 & 3

18. For the quadratic function $f(x) = x^2 - 6x + 5$, find the following and graph. **(2pts each)**

Vertex $(h, k)$ $\quad h = \frac{-(-6)}{2(1)} = \frac{6}{2} = 3$; $\quad k = f(3) = (3)^2 - 6(3) + 5 = 9 - 18 + 5 = -4$

a). Vertex $(3, -4)$

b). x-intercept(s) $(1, 0)$ & $(5, 0)$

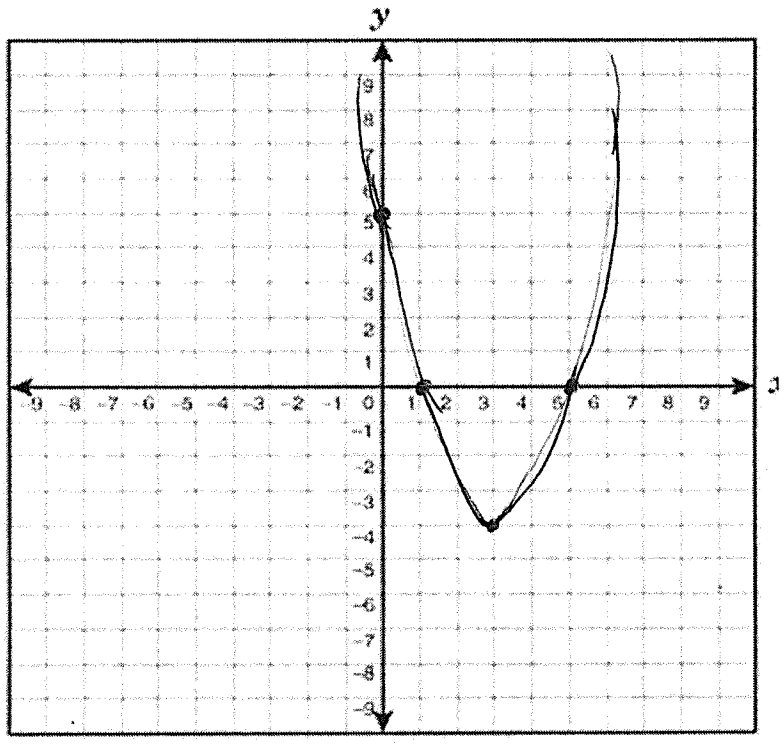
$x^2 - 6x + 5 = 0$

$(x - 5)(x - 1) = 0$

$x - 5 = 0 \qquad x - 1 = 0$

$x = 5 \qquad x = 1$

c). y-intercept 5 or $(0, 5)$




d). Graph to the right



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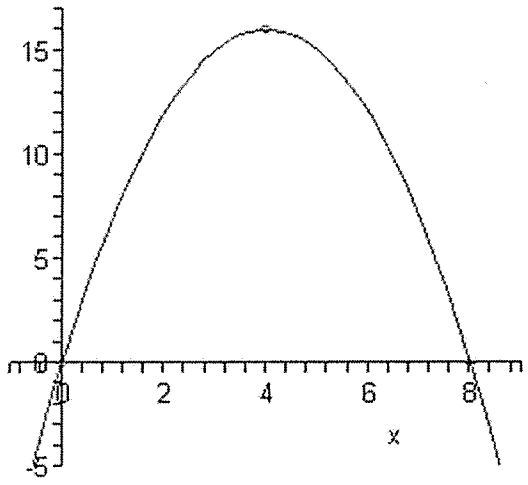
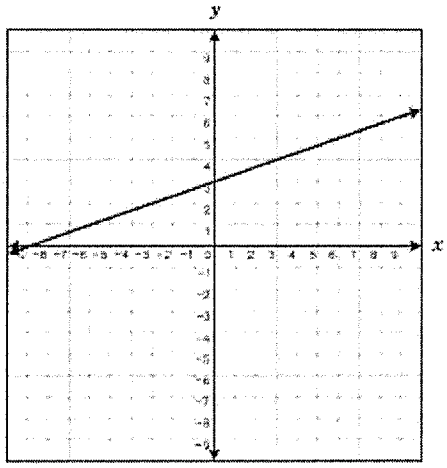
**Match the graph to the type of function that best describes it. The same type may be used multiple times or not at all. (2pts each)**

a). Linear b). Quadratic c). Exponential d). Radical e). Rational

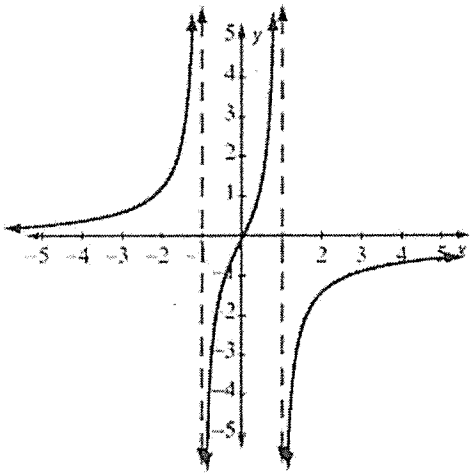
19. Quadratic (b)



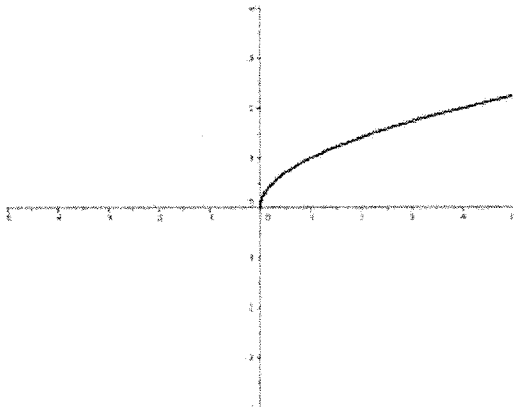

20. Linear (a)




21. Rational (e)




22. Radical (d)





8

**Solve each equation below. Simplify completely, do not round. (3 pts each)**

23. $6x^2 - 13x = 5$

$6x^2 - 13x - 5 = 0$

$(3x+1)(2x-5) = 0$

$3x+1 = 0$ or $2x-5 = 0$

$3x = -1$ $\quad 2x = 5$

$x = -1/3$ $\quad x = 5/2$

24. $x = 3 + \sqrt{x-1}$

$x - 3 = \sqrt{x-1}$

$(x-3)^2 = (\sqrt{x-1})^2$

$x^2 - 6x + 9 = x - 1$

$x^2 - 6x + 9 - x + 1 = 0$

$x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

$x - 5 = 0$ or $x - 2 = 0$

$x = 5$ or $x = 2$ (Extraneous)

only solution

25. $\frac{6}{x+1} = \frac{x}{x-1}$ ; $x \neq \pm 1$

$x(x+1) = 6(x-1)$

$x^2 + x = 6x - 6$

$x^2 + x - 6x + 6 = 0$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$x - 3 = 0$ $\quad x - 2 = 0$

$x = 3$ $\quad x = 2$

**Solve each equation below. Simplify completely, do not round. (3 pts each)**

26. $3x - \frac{2}{x} = -5$ ; $x \neq 0$

Multiply each term by $x$.

$3x^2 - 2 = -5x$

$3x^2 + 5x - 2 = 0$

$(3x - 1)(x + 2) = 0$

$3x - 1 = 0$ or $x + 2 = 0$

$3x = 1$ $\quad$ $x = -2$

$x = \frac{1}{3}$

27. $x^2 + 4x + 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)} = \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$x = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm i\sqrt{4}}{2} = \frac{-4 \pm 2i}{2}$$

$$x = -\frac{4}{2} \pm \frac{2}{2}i$$

$x = -2 \pm i$



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**Application Problems. For all problems where an equation is not given, you need to define your variable(s), set up an algebraic equation or equations, solve algebraically, and answer the question with the proper units. If an equation is given, be sure to answer the question completely and with proper units. (4 pts each)**

28. Suppose H(x) = $-x^2 + 10x + 11$ gives the height H in feet of a ball x seconds after it is kicked. How long has the ball been airborne if it is intercepted 32 feet above ground on its way back down?

$-x^2 + 10x + 11 = 32$

$0 = x^2 - 10x - 11 + 32$

$0 = x^2 - 10x + 21$

$0 = (x-7)(x-3)$

$x - 7 = 0$ or $x - 3 = 0$

$x = 7$ or $x = 3$

7 seconds

29. Suppose r(x) = $-3x^2 + 54x + 200$ gives the total revenue in dollars made by a computer repair shop for repairing x computers per day.

a). How many computers must be repaired per day to maximize revenue? (2 pts)

$x = \frac{-b}{2a} = \frac{-54}{2(-3)} = \frac{-54}{-6} = 9$

9 Computers

b). What is the maximum revenue per day? (2 pts)

$r(9) = -3(9)^2 + 54(9) + 200$

$= -3(81) + 486 + 200$

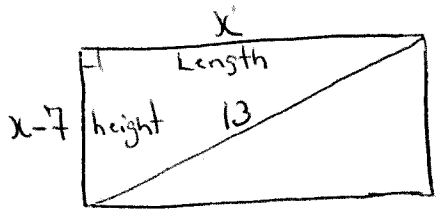
$= -243 + 486 + 200 = 443$

$443



18

30. The height of a rectangular wall is 7 feet shorter than the horizontal distance that it spans. If the wall measures 13 feet diagonally, then find the height and length of the wall.




$$x^2 + (x-7)^2 = (13)^2$$
$$x^2 + x^2 - 14x + 49 = 169$$
$$2x^2 - 14x + 49 - 169 = 0$$
$$2x^2 - 14x - 120 = 0$$
$$2(x^2 - 7x - 60) = 0$$
$$2(x-12)(x+5) = 0$$

$x - 12 = 0$ → $x = 12$

$x + 5 = 0$ → $x = -5$

Length of wall is: 12 feet

Height of wall is: 5 feet

31. Train-X travels 14 mph faster than train-Y. Suppose the train-X travels 400 miles in the same time it takes train-Y to travel 330 miles. Find the speed of each vehicle.

Speed of train-x = $x + 14$

Speed of train-y = $x$

$t$ = time in hours

| | time = $\frac{\text{distance}}{\text{speed}}$ |
|---|---|
| Train-X | $t = \frac{400}{x+14}$ |
| Train-Y | $t = \frac{350}{x}$ |

$$\frac{400}{x+14} = \frac{350}{x}$$
$$400x = 350(x+14)$$
$$400x = 350x + 4900$$
$$400x - 350x = 4900$$
$$50x = 4900$$
$$x = 98$$

Train-X's speed: 112 miles per hour

Train-Y's speed: 98 miles per hour

32. Ellen can rake her yard twice as fast as her husband, Peter. If they work together, it takes them 3 hours to rake the yard. How long would it take Peter to rake the entire yard alone?

Ellen's time = $x$ hrs.

Peter's time = $2x$ hrs.

$$\frac{1}{x} + \frac{1}{2x} = \frac{1}{3} \quad ; \quad LCD = 6x$$

$$6x\left(\frac{1}{x} + \frac{1}{2x}\right) = 6x\left(\frac{1}{3}\right)$$

$$6 + 3 = 2x$$

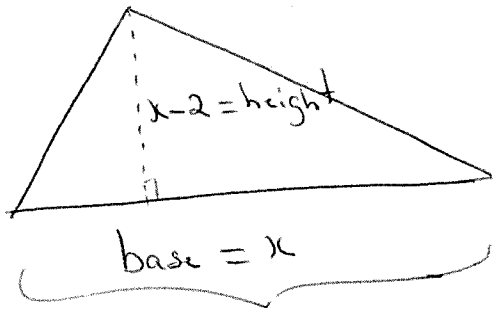
$$9 = 2x$$

$x = \frac{9}{2}$ hours

Ellen: $\frac{9}{2}$ hours.

Peter: 9 hours

33. The height of a triangular coffee field is 2 miles shorter than its base. If the surface area of the field is 12 square miles, then find the base and height of the coffee field.




$$\frac{x(x-2)}{2} = 12$$

$$x(x-2) = 24$$

$$x^2 - 2x = 24$$

$$x^2 - 2x - 24 = 0$$

$$(x-6)(x+4) = 0$$

$x - 6 = 0$ $\qquad$ $x + 4 = 0$

$x = 6$ $\qquad$ $x \neq -4$

Base of coffee field is: 6 miles

Height of coffee field is: 4 miles



18

**Bonus (3pts) –No partial credit!**

Suppose Jenny has 6 shirts, 9 skirts, and 4 hats. If an outfit consists of one shirt, one skirt, and one hat, how many different outfits can Jenny create?

$(6)(9)(4) = 216$ outfits

*Have a Wonderful Summer Recess!*