

Name: _____

KEY

Date: _____

Intermediate Algebra Final Exam: You have two hours to complete this exam. You may use a calculator (TI-84 or lower), but you may not use a cell phone, your book, the internet, or any other notes. You must show all work to earn credit, and all answers must be clearly marked and in simplest form.

Section 1: Factor each polynomial completely. If the polynomial cannot be factored, you must explicitly state this. (2 points each)

1) $6x^2 + 7x + 2$

$$6x^2 + 3x + 4x + 2$$

$$3x(2x+1) + 2(2x+1)$$

$$(2x+1)(3x+2)$$

3) $x^2 - x - 30$

$$(x-6)(x+5)$$

2) $49x^2 - 4$

$$(7x+2)(7x-2)$$

4) $5x^3 + 20x^2 + 20x$

$$5x[x^2 + 4x + 4]$$

$$5x(x+2)(x+2)$$

or

$$5x(x+2)^2$$

Section 2: Simplify each expression completely. Perform any indicated operation, rationalize all denominators, and leave complex answers in the form: $a + bi$. (3 points each)

5) $\frac{x^2+5x+6}{x^2+7x+10} \cdot \frac{x^2+2x-35}{x^2-2x-15}$

$$\frac{(x+2)(x+3)}{(x+2)(x+5)} \cdot \frac{(x+7)(x-5)}{(x-5)(x+3)}$$

$$\boxed{\frac{x+7}{x+5}}$$

6) $\frac{7x-3}{x^2-4} + \frac{6x-1}{4-x^2}$

$$\frac{7x-3}{x^2-4} - \frac{6x-1}{x^2-4}$$

$$\frac{x-2}{x^2-4}$$

$$\frac{x-2}{(x+2)(x-2)}$$

$$= \boxed{\frac{1}{x+2}}$$

7) $\left(\frac{7}{10y}\right)^3 - \left(\frac{4}{15y}\right)^2$

$$\frac{21}{30y} - \frac{8}{30y}$$

$$= \boxed{\frac{13}{30y}}$$

Section 2: Simplify each expression completely. Perform any indicated operation, rationalize all denominators, and leave complex answers in the form: $a + bi$. (3 points each)

8) $2\sqrt{75} + \sqrt{3}$

$$2 \cdot 5\sqrt{3} + \sqrt{3}$$

$$10\sqrt{3} + \sqrt{3}$$

$$\boxed{11\sqrt{3}}$$

11) $(3 + \sqrt{2})(3 - \sqrt{2})$

$$9 - 3\sqrt{2} + 3\sqrt{2} - \sqrt{4}$$

$$9 - 2$$

$$\boxed{7}$$

9) $\sqrt[5]{32x^{10}y^{27}}$

$$\boxed{2x^2y^5\sqrt[5]{y^2}}$$

12) $4i(2 - i)$

$$8i - 4i^2$$

$$8i - 4(-1)$$

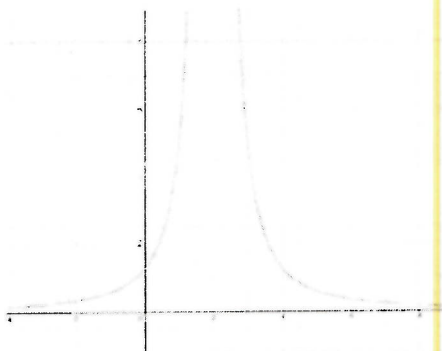
$$\boxed{4 + 8i}$$

10) $\frac{1}{\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}} \right)$

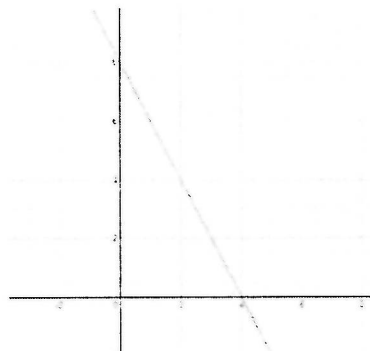
$$\boxed{\frac{\sqrt{2}}{2}}$$

Section 3: Match the graph to the type of function that best describes it. The same type may be used multiple times or not at all. (2 points each)

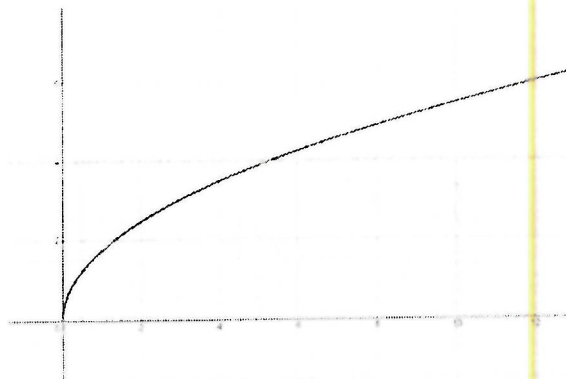
- a) Linear b) Quadratic c) exponential d) Radical e) Rational



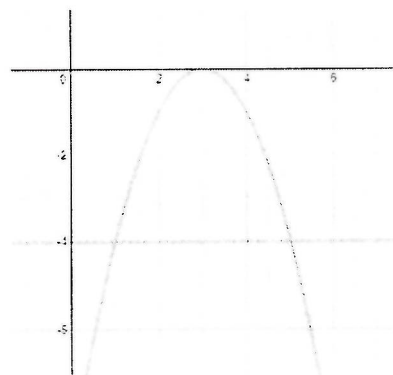
13) e) Rational



15) a) Linear



14) d) Radical



16) b) Quadratic

Section 4: List the domain or any restrictions on the domain for the following functions. (2 points each)

17) $f(x) = \sqrt{4x - 36}$

$$4x - 36 \geq 0$$

$$4x \geq 36$$

$$x \geq 9$$

$$\text{Domain: } \{x \mid x \geq 9\}$$

18) $G(x) = x^3 + 2x^2 + 6x + 12$

No Restrictions

$$\text{Domain: } \{x \mid \text{all real numbers}\}$$

19) $T(x) = \frac{x+1}{x^2-9}$

$$x^2 - 9 \neq 0$$

$$(x+3)(x-3) \neq 0$$

$$x+3 \neq 0, \quad x-3 \neq 0$$

$$x \neq -3, \quad x \neq 3$$

$$\text{Domain: } \{x \mid x \neq 3, -3\}$$

Section 5: Find the following points and use them to graph the given quadratic. (2 points each)

20) Given: $y = x^2 - 8x + 7$

$y = (x-1)(x-7)$

$0 = x-1$, $0 = x-7$

$x=1$, $x=7$

Vertex: $x = \frac{-b}{2a}$

$x = \frac{8}{2(1)} = 4$

$y = 4^2 - 8(4) + 7$

$y = -9$

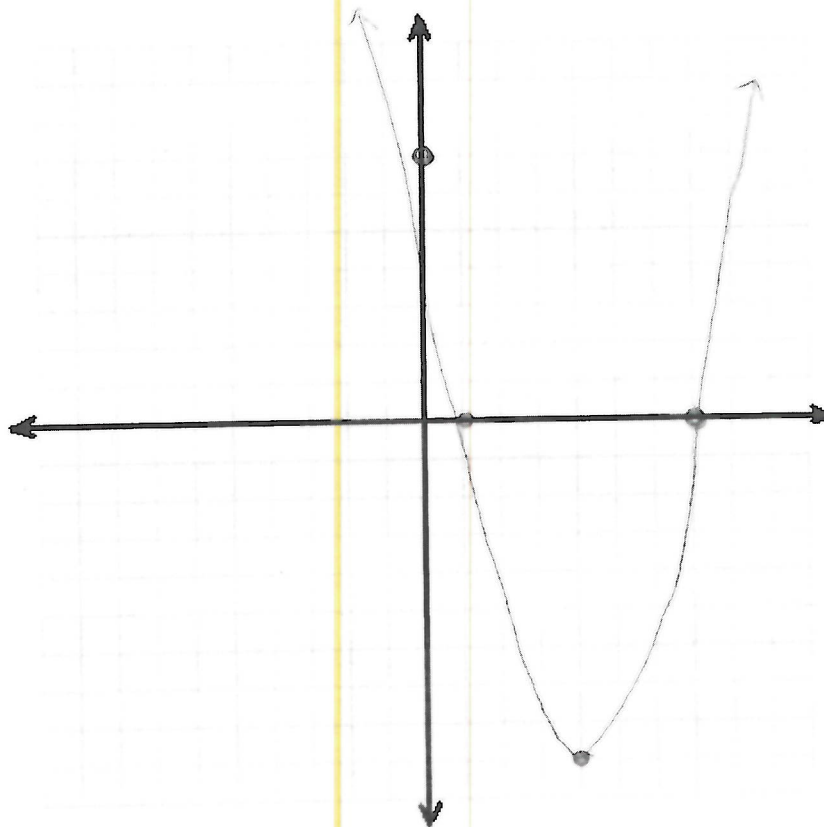
a. Find the y - intercept: (0, 7)

b. Find the x - intercept(s): (1, 0) and (7, 0)

c. Find the vertex: (4, -9)

d. Does this function have a Maximum, Minimum, or Neither ?

e. Graph:



Section 6: Solve the following equations for the given variable. If there is no solution, you must explicitly state that is the case (3 points each)

21) $7 + \sqrt{2x - 3} = 12$

$$\begin{array}{r} -7 \quad -7 \\ \hline (\sqrt{2x-3})^2 = (5)^2 \\ 2x-3 = 25 \\ +3 \quad +3 \\ \hline 2x = 28 \\ \frac{2x}{2} = \frac{28}{2} \\ \hline \end{array}$$

$x = 14$

22) $\frac{5}{x+4} = \frac{3}{x}$ LCD = $x(x+4)$
cross multiply

$$\begin{array}{r} 5(x) = 3(x+4) \\ 5x = 3x + 12 \\ -3x \quad -3x \\ \hline 2x = 12 \\ \frac{2x}{2} = \frac{12}{2} \\ \hline \end{array}$$

$x = 6$

Section 6: Solve the following equations for the given variable. If there is no solution, you must explicitly state that is the case (3 points each)

23) $3x^2 + 6 = 11x$

$$3x^2 - 11x + 6 = 0$$

↓ ↓

$$3x^2 - 9x - 2x + 6 = 0$$

$$3x(x-3) - 2(x-3) = 0$$

$$(x-3)(3x-2) = 0$$

$$x-3=0, \quad 3x-2=0$$

$$3x=2$$

$$x=3, \quad x=\frac{2}{3}$$

24) $x^2 - 4x + 5 = 0$

$$a=1$$

$$b=-4$$

$$c=5$$

$$x = \frac{+4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16-20}}{2}$$

$$x = \frac{4 \pm \sqrt{-4}}{2}$$

$$x = \frac{4 \pm 2i}{2}$$

$$x = 2 \pm i$$

Section 6: Solve the following equations for the given variable. If there is no solution, you must explicitly state that is the case (3 points each)

25) $\frac{3\sqrt[4]{x}}{3} = 6$
 $(\sqrt[4]{x}) (2)$
 $x = 16$

26) $\text{LCD} \left(\frac{x}{3x-12} + \frac{2}{x-4} \right) = 2 \text{ (us)}$
 $3x-12 = 3(x-4)$
 $\text{LCD} = 3(x-4)$

$x + 3(2) = 2(3(x-4))$
 $x + 6 = 6x - 24$
 $-x \quad -x$
 $6 = 5x - 24$
 $+24 \quad +24$
 $30 = 5x$
 $\frac{30}{5} = \frac{5x}{5}$
 $x = 6$

Section 6: Solve the following equations for the given variable. If there is no solution, you must explicitly state that is the case (3 points each)

27) $5x^2 - x = 2$

$$5x^2 - x - 2 = 0$$

$$a = 5$$

$$b = -1$$

$$c = -2$$

$$x = \frac{+1 \pm \sqrt{(-1)^2 - 4(5)(-2)}}{2(5)}$$

$$x = \frac{1 \pm \sqrt{41}}{10}$$

28) $(\sqrt{x-2})^2 = 8-x$

$$x-2 = (8-x)(8-x)$$

$$x-2 = 64 - 16x + x^2$$

$$0 = 66 - 17x + x^2$$

$$0 = x^2 - 17x + 66$$

$$0 = (x-11)(x-6)$$

$$x = 11$$

$$x = 6$$

check: $x = 11$

$$\sqrt{11-2} = 8-11$$

$$\sqrt{9} = -3$$

$$3 = -3 \quad \times$$

check: $x = 6$

$$\sqrt{6-2} = 8-6$$

$$\sqrt{4} = 2$$

$$2 = 2 \quad \checkmark$$

Section 7: Solve the following word problems. Solutions must include units and show all algebraic work to receive full credit. (4 points each)

- 29) The bookstore wants to maximize its revenue from the sale of TI-30 calculators. Their Revenue function can be approximated by

$$R(x) = -150x^2 + 900x + 600$$

where $R(x)$ is the revenue in dollars from selling x hundred calculators. How many calculators should they sell to maximize their revenue and what will the revenue be?

① Calculators = x

② Maximize \rightarrow Vertex: $x = \frac{-b}{2a}$

③ $x = \frac{-900}{2(-150)} = \frac{-900}{-300} = 3$

$$R(3) = -150(3)^2 + 900(3) + 600$$

$$R(3) = 1950$$

④ They should sell 300 calculators to make \$1950 in Revenue

- 30) A park ranger captures and tags 84 foxes from the forest. Later, 45 foxes are captured and 7 of them have tags. How many foxes are there in the entire forest?

① Foxes = x

② Ratios: $\frac{7}{45} = \frac{84}{x}$

③ $\frac{7x}{7} = \frac{3780}{7}$

$$x = 540$$

④ There are 540 foxes

Section 7: Solve the following word problems. Solutions must include units and show all algebraic work to receive full credit. (4 points each)

31) The area of a triangle is 27 square meters. The base of the triangle is 3 meters longer than its height. Find the height and base of the triangle.

$$\textcircled{1} \begin{aligned} \text{height} &= x \\ \text{base} &= x+3 \end{aligned}$$

$$\textcircled{2} 2A = b \cdot h$$

$$\textcircled{3} 2(27) = (x+3)x$$

$$54 = x^2 + 3x$$

$$0 = x^2 + 3x - 54$$

$$0 = (x+9)(x-6)$$

$$\begin{aligned} x+9 &= 0 & x-6 &= 0 \\ x &= -9 & x &= 6 \end{aligned}$$

$\textcircled{4}$ The height is 6 m
the base is 9 m

32) Two pumps are being used to fill the school's swimming pool. The new pump works four times as fast as the old pump. If it takes 4 hours to fill the swimming pool using both pumps, how long would it take the new pump working alone?

$$\textcircled{1} \text{ New pump time} = x$$

$$\textcircled{2} \frac{1}{A} + \frac{1}{B} = \frac{1}{T}$$

$$\textcircled{3} \frac{1}{x} + \frac{1}{4x} = \frac{1}{4} \quad \text{Len} = 4x$$

$$4 + 1 = x$$

$$5 = x$$

$\textcircled{4}$ It would take the new pump 5 hours alone.

Section 7: Solve the following word problems. Solutions must include units and show all algebraic work to receive full credit. (4 points each)

- 33) Tom and Brad are driving home separately from the store. Brad drives 10 mph faster than Tom. Tom covers 72 miles in the same amount of time Brad covers 96 miles. How fast is Tom driving and how fast is Brad driving?

(1) Tom Speed = x
Brad's Speed = $x + 10$

(2)

	d	r	t
Tom	72	x	$\frac{72}{x}$
Brad	96	$x + 10$	$\frac{96}{x + 10}$

(3) $\frac{72}{x} = \frac{96}{x + 10}$

$72(x + 10) = 96(x)$

$72x + 720 = 96x$
 $-72x$ $-72x$

$720 = 24x$
 $\frac{720}{24} = \frac{24x}{24}$

$30 = x$

(4)

Extra Credit: 4 possible point. NO CREDIT FOR GUESSING

In a right triangle where n and m are positive numbers, the sides have lengths: n , $n + m$, and $n + 2m$. What is the value of $\frac{n}{m}$?



$a^2 + b^2 = c^2$

$(n)^2 + (n + m)^2 = (n + 2m)^2$

$n^2 + n^2 + 2nm + m^2 = n^2 + 4nm + 4m^2$

$n^2 - 2nm - 3m^2 = 0$

$(n + m)(n - 3m) = 0$

$\cancel{n + m = 0}$ $n - 3m = 0$

$\frac{n}{m} = \frac{3m}{m}$

$\frac{n}{m} = 3$