Intermediate Algebra Final Exam: You have two hours to complete this exam. You may use a calculator (TI-84 or lower), but you may not use a cell phone, your book, the internet, or any other notes. You must show all work to earn credit, and all answers must be clearly marked and in simplest form.

Section 1: Factor each polynomial completely. If the polynomial cannot be factored, you must explicitly state this. (2 points each)

1) \( 3x^2 + 8x + 4 \)
   \[
   \begin{align*}
   &3x^2 + 6x + 2x + 4 \\
   &3x(x+2) + 2(x+2) \\
   &\boxed{(x+2)(3x+2)}
   \end{align*}
   \]

2) \( 9x^2 - 25 \)
   \[
   \boxed{(3x+5)(3x-5)}
   \]

3) \( x^2 - 11x + 18 \)
   \[
   \boxed{(x-2)(x-9)}
   \]

4) \( 3x^7 + 21x^6 + 30x^5 \)
   \[
   \boxed{3x^5(x+2)(x+5)}
   \]
Section 2: Simplify each expression completely. Perform any indicated operation, rationalize all denominators, and leave complex answers in the form: \( a + bi. \) (3 points each)

5) \( \frac{2x+3}{3x+9} \div \frac{2x^2-7x-15}{3x-15} \)

\[ \frac{2x+3}{3x+9} \cdot \frac{3x-15}{2x^2-7x-15} \]

\[ \frac{(2x+3)(x-5)}{3(x+3)(2x+3)(x+5)} \]

\[ \frac{1}{x+3} \]

6) \( \frac{7x-3}{5x-2} + \frac{2x-1}{2-5x} \)

\[ 2-5x = -1(5x-2) \]

\[ \frac{7x-3}{5x-2} - \frac{2x-1}{5x-2} \]

\[ \frac{5x-2}{5x-2} \]

\[ 1 \]

7) \( \frac{3}{5x} - \frac{4}{x^2} \)

\[ \text{LCM} = 5x^2 \]

\[ \frac{3x-20}{5x^2} \]
Section 2: Simplify each expression completely. Perform any indicated operation, rationalize all denominators, and leave complex answers in the form: \( a + bi \). (3 points each)

8) \( \sqrt{50} - \sqrt{18} \)

\[
5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}
\]

9) \( \sqrt[3]{27x^{12}y^{17}} \)

\[
3x^4y^5\sqrt[3]{y^2}
\]

10) \( \sqrt[3]{\frac{2}{3}} \left( \frac{3}{3} \right) \)

\[
\frac{\sqrt[3]{6}}{\sqrt[3]{9}} = \frac{\sqrt[3]{6}}{3}
\]

11) \( 3\sqrt[5]{4 + \sqrt{15}} \)

\[
\frac{12\sqrt{5} + 3\sqrt{75}}{12\sqrt{5} + 15\sqrt{3}}
\]

12) \( \frac{\frac{3}{1+i}}{1-i} = \frac{3-3i}{1-i^2} \)

\[
\frac{3-3i}{2} = \frac{3}{2} - \frac{3}{2}i
\]
Section 3: Match the graph to the type of function that best describes it. The same type may be used multiple times or not at all. (2 points each)

a) Linear       b) Quadratic       c) exponential       d) Radical       e) Rational

13) a) Linear

14) e) Rational

15) b) Quadratic

16) d) Radical
Section 4: List the domain or any restrictions on the domain for the following functions. (2 points each)

17) \( f(x) = \frac{5x-2}{2x-5} \)

\[\begin{align*}
&2x-5 \neq 0 \\
&2x \neq 5 \\
\text{Domain: } &\{x \mid x \neq \frac{5}{2}\}
\end{align*}\]

18) \( G(x) = 2 + \sqrt{3x - 15} \)

\[\begin{align*}
&3x - 15 \geq 0 \\
&3x \geq 15 \\
\text{Domain: } &\{x \mid x \geq 5\}
\end{align*}\]

19) \( T(x) = 3x^2 + 8x + 15 \)

\[\text{Domain: } \{x \mid \text{all real numbers}\}\]
Section 5: Find the following points and use them to graph the given quadratic. (2 points each)

20) Given: \( y = x^2 + 2x - 8 \)

\[ \frac{-b}{2a} = \frac{-2}{2} = -1 \]

\[ y = (-1)^2 + 2(-1) - 8 = -9 \]

\[ x = \frac{-b}{2a} = \frac{-2}{2} = -1 \]

\[ a = 1, b = 2 \]

a. Find the \( y \) - intercept: \((0, -8)\)

b. Find the \( x \) - intercept(s): \((-4, 0)\) and \((2, 0)\)

c. Find the vertex: \((-1, -9)\)

d. Does this function have a Maximum, Minimum, or Neither?

e. Graph:
Section 6: Solve the following equations for the given variable. If there is no solution, you must explicitly state that is the case (3 points each)

21) \[ 3x^2 - 5x = 2 \]

\[ 3x^2 - 5x - 2 = 0 \]
\[ 3x^2 - 6x + x - 2 = 0 \]
\[ 3x(x - 2) + 1(x - 2) = 0 \]
\[ (x - 2)(3x + 1) = 0 \]

\[ x = 2, \ -\frac{1}{3} \]

22) \[ \frac{7}{3} = \frac{x + 2}{x - 2} \]

\[ 7(x - 2) = 3(x + 2) \]
\[ 7x - 14 = 3x + 6 \]
\[ -3x + 14 = 3x + 14 \]

\[ 4x = 20 \]

\[ x = 5 \]
Section 6: Solve the following equations for the given variable. If there is no solution, you must explicitly state that is the case (3 points each)

23) \[ 10 = \sqrt[3]{x} + 5 \]

\[ (5)^3 - (\sqrt[3]{x})^3 \]

\[ x = 125 \]

24) \[ x^2 + 10 = 2x \]

\[ x^2 - 2x + 10 = 0 \]

\[ a = 1 \]
\[ b = -2 \]
\[ c = 10 \]

\[ x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(10)}}{2(1)} \]

\[ x = \frac{2 \pm \sqrt{-36}}{2} \]

\[ x = \frac{2 \pm 6i}{2} \]

\[ x = 1 \pm 3i \]
Section 6: Solve the following equations for the given variable. If there is no solution, you must explicitly state that is the case (3 points each)

25) \[ \frac{1}{x} + \frac{3}{2} = \frac{11}{2x} \]

\[ 2 + 3x = 11 \]
\[ 3x = 9 \]
\[ x = 3 \]

26) \[ \sqrt{x+28} = (x - 2)^2 \]
\[ x + 28 = x^2 - 4x + 4 \]
\[ -x - 2x = -x - 28 \]
\[ 0 = x^2 - 5x - 24 \]
\[ 0 = (x-8)(x+3) \]
\[ x = 8, x = -3 \]

Check:
\[ x = 8: \]
\[ \sqrt{8+28} = 8 - 2 \]
\[ 6 = 6 \sqrt{ } \]

\[ x = -3: \]
\[ \sqrt{-3+28} = -3 - 2 \]
\[ 5 = 5 \sqrt{ } \]
Section 6: Solve the following equations for the given variable. If there is no solution, you must explicitly state that is the case (3 points each)

27) \[ \frac{-4}{x-4} = 3 - \frac{x}{x-4} \]

\[ -4 = 3(x-4) - x \]
\[ -4 = 3x - 12 - x \]
\[ -4 = 2x - 12 \]
\[ 8 = 2x \]
\[ x = 4 \]

Check \( x = 4 \)
\[ \frac{-4}{0} = 3 - \frac{4}{0} \text{ is undefined} \]

No Solutions

28) \( 7x^2 + 3x - 1 = 0 \)

\[ a = 7 \]
\[ b = 3 \]
\[ c = -1 \]

\[ x = \frac{-3 \pm \sqrt{(3)^2 - 4(7)(-1)}}{2(7)} \]

\[ x = \frac{-3 \pm \sqrt{37}}{14} \]
Section 7: Solve the following word problems. Solutions must include units and show all algebraic work to receive full credit. (4 points each)

29) After their final exam, Brian and Alex both leave campus at the same time. Brian drives 8 mph faster than Alex, and at one point, Brian has driven 12 miles and Alex has driven 10 miles. How fast was Brian driving?

\[ \text{Brians speed} = x \]

<table>
<thead>
<tr>
<th>Brian</th>
<th>12</th>
<th>x</th>
<th>\frac{12}{x}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alex</td>
<td>10</td>
<td>x-8</td>
<td>\frac{10}{x-8}</td>
</tr>
</tbody>
</table>

Same time: \[ \frac{12}{x} = \frac{10}{x-8} \]
\[ 12(x-8) = 10x \]
\[ 12x - 96 = 10x \]
\[ -96 = -2x \]
\[ 48 = x \]

30) A woodworker is making the bottom of a rectangular box with an area of 40 square inches. The width of the box is 3 inches more than its length. Find the length and the width of the box.

\[ \text{length} = x \]
\[ \text{width} = x + 3 \]
\[ lw = A \]
\[ x(x+3) = 40 \]
\[ x^2 + 3x - 40 = 0 \]
\[ (x+8)(x-5) = 0 \]
\[ x = 8 \quad x = 5 \]

The length is 5 inches.
The width is 8 inches.
Section 7: Solve the following word problems. Solutions must include units and show all algebraic work to receive full credit. (4 points each)

29) James and Nathan are electricians wiring a house. It would take James 9 hours if he was working alone, and it would take Nathan 18 hours if he was working alone. How long will it take them working together?

\[ \text{Together time} = x \]
\[ \frac{1}{9} + \frac{1}{18} = \frac{1}{x} \]
\[ \text{LCM} = 18 \times \]
\[ 2x + x = 18 \]
\[ 3x = 18 \]
\[ x = 6 \]

It will take 6 hrs working together

30) Andre owns a bakery that makes apple pies. He records data about his sales when he changes the price of his apple pies. He is able to model his weekly profit \( P \) as a function of the price he charges for each pie as:

\[ P(x) = -2x^2 + 52x - 38 \]

where \( x \) = price per pie.

What price should he charge per pie to maximize profit? What is his maximum weekly profit?

\[ a = -2, \ b = 52 \]
\[ x = \frac{-b}{2a} = \frac{-52}{-4} = 13 \]

\[ P(13) = -2(13)^2 + 52(13) - 38 \]
\[ -338 + 676 - 38 \]
\[ P(13) = 300 \]

He should charge $13 and will make $1300 weekly profit.
Section 7: Solve the following word problems. Solutions must include units and show all algebraic work to receive full credit. (4 points each)

31) I launch a model rocket upwards at a speed of 112 feet per second. The height \( h(t) \) in meters after \( t \) seconds is given by the equation:

\[
h(t) = -16t^2 + 112t
\]

When will the rocket hit the ground?

When \( h(t) = 0 \):

\[
0 = -16t^2 + 112t
0 = -16t(t - 7)
\]

When \( t = 0 \), the rocket hits the ground after 7 seconds.

Extra Credit: 4 possible point. NO CREDIT FOR GUESSING

Justin must drive from Anytown to Somicity. He can drive 10 miles on a rural road straight there at 40mph. Or he can drive east then north on highways at 60mph. The alternative route forms a right angle (see picture). The eastern leg is 2 miles longer than the northern leg. Which route will get him to Somicity quicker?

\[
x^2 + (x^2 + 2)^2 = 10^2
\]

\[
x^2 + x^2 + 4x + 4 = 100
2x^2 + 4x - 96 = 0
2(x^2 + 2x - 48) = 0
2(x + 8)(x - 6) = 0
x = 8, x = 6
\]

Rural:

\[
d = 10, \quad r = 40
\]

\[
\ell = \frac{10}{40} = \frac{1}{4} \text{ hr} = .25 \text{ hr}
\]

Highways:

\[
d = 6 + 8 = 14, \quad r = 60
\]

\[
\ell = \frac{14}{60} = \frac{7}{30} \text{ hr} = .23 \text{ hr}
\]

The highways are faster.