Tunxis Community College – Fall 2018 XXXX. Spring 2019 Sample - Answers

Name: KEY _____ **Date:** _____

Intermediate Algebra Final Exam: You have two hours to complete this exam. You may use a calculator (TI-84 or lower), but you may not use a cell phone, your book, the internet, or any other notes. You must show all work to earn credit, and all answers must be clearly marked and in simplest form.

Section 1: Factor each polynomial completely. If the polynomial cannot be factored, you must explicitly state this. *(2 points each)*

1) $3x^2 + 8x + 4$

$3x^2+6x+2x+4$

$3x(x+2)+2(x+2)$

$(x+2)(3x+2)$

2) $9x^2 - 25$

$(3x+5)(3x-5)$

3) $x^2 - 11x + 18$

$(x-2)(x-9)$

4) $3x^7 + 21x^6 + 30x^5$

$3x^5[x^2+7x+10]$

$3x^5(x+2)(x+5)$

8

Section 2: Simplify each expression completely. Perform any indicated operation, rationalize all denominators, and leave complex answers in the form: $a + bi$. *(3 points each)*

5) $\frac{2x+3}{3x+9} \div \frac{2x^2-7x-15}{3x-15}$

$\frac{2x+3}{3x+9} \cdot \frac{3x-15}{2x^2-7x-15}$

$\frac{(2x+3)}{3(x+3)} \frac{3(x-5)}{(2x+3)(x+5)}$

$\frac{1}{x+3}$

6) $\frac{7x-3}{5x-2} + \frac{2x-1}{2-5x}$ $\quad 2-5x = -1(5x-2)$

$\frac{7x-3}{5x-2} - \frac{2x-1}{5x-2}$

$\frac{5x-2}{5x-2}$

1

7) $\frac{3}{5x} - \frac{4}{x^2}$ $\quad LCD = 5x^2$

$\frac{3x-20}{5x^2}$

9

Section 2: Simplify each expression completely. Perform any indicated operation, rationalize all denominators, and leave complex answers in the form: $a + bi$. *(3 points each)*

8) $\sqrt{50} - \sqrt{18}$

$5\sqrt{2} - 3\sqrt{2}$

$\boxed{2\sqrt{2}}$

9) $\sqrt[3]{27x^{12}y^{17}}$

$\boxed{3x^4 y^5 \sqrt[3]{y^2}}$

10) $\sqrt{\frac{2}{3}}\left(\frac{3}{3}\right)$

$\frac{\sqrt{6}}{\sqrt{9}} = \boxed{\frac{\sqrt{6}}{3}}$

11) $3\sqrt{5}(4 + \sqrt{15})$

$12\sqrt{5} + 3\sqrt{75}$

$\boxed{12\sqrt{5} + 15\sqrt{3}}$

12) $\left(\frac{3}{1+i}\right)\left(\frac{1-i}{1-i}\right) = \frac{3-3i}{1-i^2}$

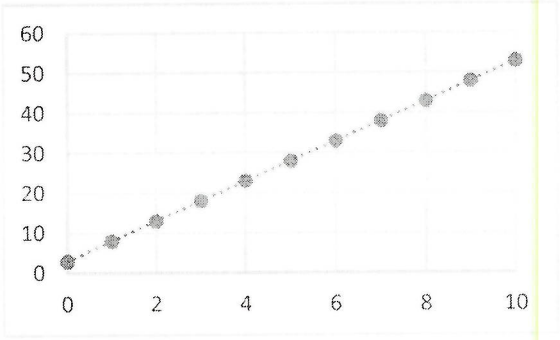
$\frac{3-3i}{2}$

$\boxed{\frac{3}{2} - \frac{3}{2}i}$

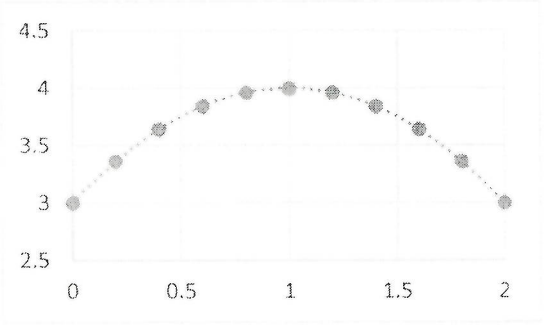
15

Section 3: Match the graph to the type of function that best describes it. The same type may be used multiple times or not at all. *(2 points each)*

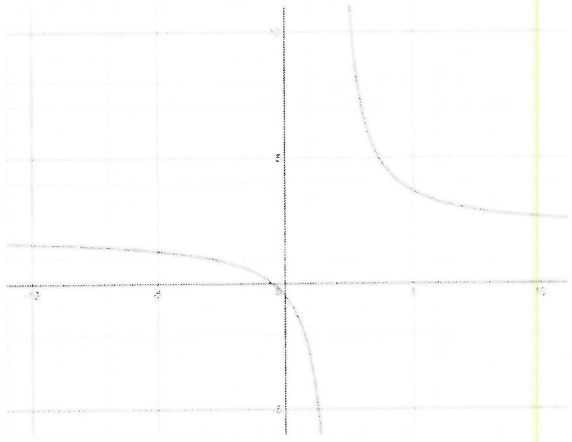
a) Linear b) Quadratic c) exponential d) Radical e) Rational



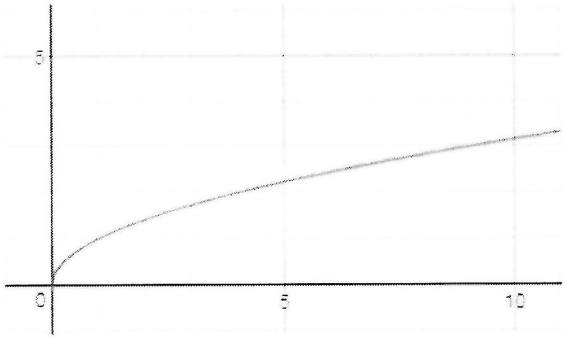
13) a) Linear



15) b) Quadratic



14) e) Rational



16) d) Radical

8

Section 4: List the domain or any restrictions on the domain for the following functions. *(2 points each)*

17) $f(x) = \frac{5x-2}{2x-5}$

$2x-5 \neq 0$
$2x \neq 5$

Domain: $\{x \mid x \neq \frac{5}{2}\}$

18) $G(x) = 2 + \sqrt{3x-15}$

$3x-15 \geq 0$
$3x \geq 15$

Domain: $\{x \mid x \geq 5\}$

19) $T(x) = 3x^2 + 8x + 15$

Domain: $\{x \mid$ all real numbers $\}$

Section 5: Find the following points and use them to graph the given quadratic. *(2 points each)*

20) Given: $y = x^2 + 2x - 8$

$(x+4)(x-2)$

Vertex: $a=1, b=2$

$x = \frac{-b}{2a} = \frac{-2}{2} = -1$

$y = (-1)^2 + 2(-1) - 8 = -9$

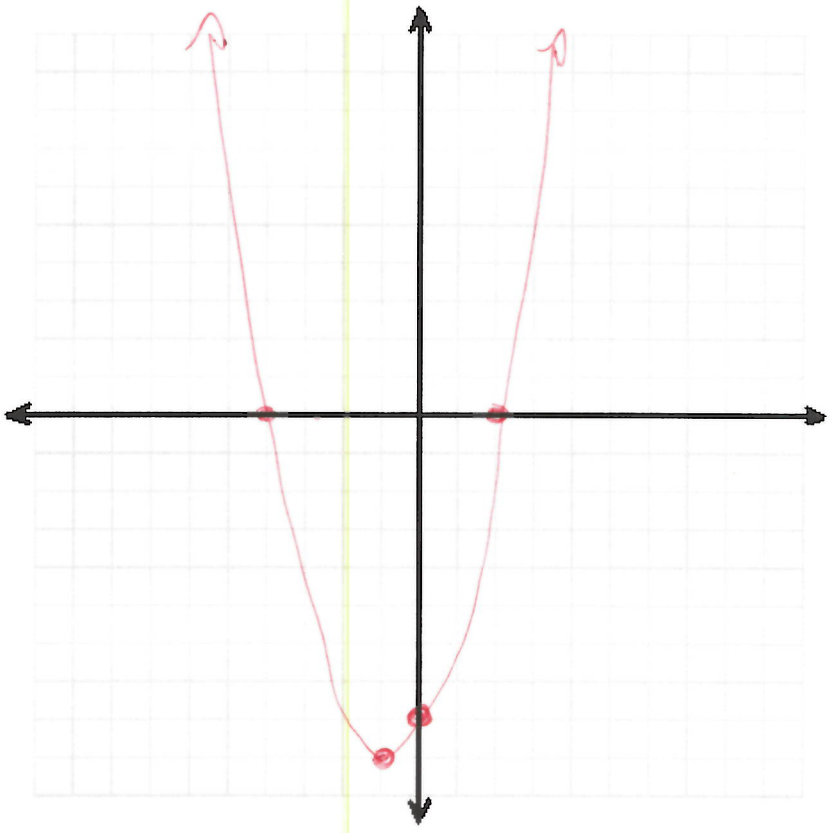
a. Find the $y-intercept$: $(0, -8)$

b. Find the $x-intercept(s)$: $(-4, 0)$ and $(2, 0)$

c. Find the $vertex$: $(-1, -9)$

d. Does this function have a Maximum, Minimum, or Neither ?

e. Graph:



10

Section 6: Solve the following equations for the given variable. If there is no solution, you must explicitly state that is the case *(3 points each)*

21) $3x^2 - 5x = 2$

$3x^2 - 5x - 2 = 0$

$3x^2 - 6x + x - 2 = 0$

$3x(x-2) + 1(x-2) = 0$

$(x-2)(3x+1) = 0$

$x = 2, -\frac{1}{3}$

22) $\frac{7}{3} = \frac{x+2}{x-2}$

$7(x-2) = 3(x+2)$

$7x - 14 = 3x + 6$

$-3x \quad +14 \quad -3x \quad +14$

$4x = 20$

$x = 5$

6

Section 6: Solve the following equations for the given variable. If there is no solution, you must explicitly state that is the case *(3 points each)*

23) $10 = \sqrt[3]{x} + 5$

$-5 \quad -5$

$(5)^3 (\sqrt[3]{x})^3$

$X = 125$

24) $x^2 + 10 = 2x$

$x^2 - 2x + 10 = 0$

$a = 1$

$b = -2$

$c = 10$

$x = \dfrac{+2 \pm \sqrt{(-2)^2 - 4(1)(10)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{-36}}{2}$

$x = \dfrac{2 \pm 6i}{2}$

$x = 1 \pm 3i$

6

Section 6: Solve the following equations for the given variable. If there is no solution, you must explicitly state that is the case *(3 points each)*

25) $\frac{1}{x} + \frac{3}{2} = \frac{11}{2x}$

LCD = 2x

$2 + 3x = 11$

$3x = 9$

$\boxed{x = 3}$

26) $\sqrt{x+28} = x - 2$

$(\sqrt{x+28})^2 = (x-2)^2$

$x + 28 = x^2 - 4x + 4$

$-x \quad -28 \qquad -x \quad -28$

$0 = x^2 - 5x - 24$

$0 = (x-8)(x+3)$

$\boxed{x = 8}$, $x = \cancel{-3}$

Check

$x = 8$:

$\sqrt{8+28} = 8 - 2$

$\sqrt{36} = 6$

$6 = 6$ ✓

$x = -3$

$\sqrt{-3+28} = -3 - 2$

$\sqrt{25} = -5$ X

6

Section 6: Solve the following equations for the given variable. If there is no solution, you must explicitly state that is the case *(3 points each)*

27) $\frac{-4}{x-4} = 3 - \frac{x}{x-4}$

LCD = $x-4$

$-4 = 3(x-4) - x$

$-4 = 3x - 12 - x$

$-4 = 2x - 12$

$+12 \quad +12$

$\frac{8}{2} = \frac{2x}{2}$

$4 \neq x$ Doesn't check

Check $x=4$

$\frac{-4}{0} = 3 - \frac{4}{0}$ ← undefined

No Solutions

28) $7x^2 + 3x - 1 = 0$

$a = 7$

$b = 3$

$c = -1$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(7)(-1)}}{2(7)}$$

$$x = \frac{-3 \pm \sqrt{37}}{14}$$

Section 7: Solve the following word problems. Solutions must include units and show all algebraic work to receive full credit. *(4 points each)*

29) After their final exam, Brian and Alex both leave campus at the same time. Brian drives 8 mph faster than Alex, and at one point, Brian has driven 12 $miles$ and Alex has driven 10 $miles$. How fast was Brian driving?

Brians Speed = x

	d	r	t
Brian	12	x	$\frac{12}{x}$
Alex	10	$x-8$	$\frac{10}{x-8}$

Brian was driving 48 mph

Same time: $\frac{12}{x} = \frac{10}{x-8}$

$12(x-8) = 10x$

$12x - 96 = 10x$

$-96 = -2x$

$48 = x$

30) A woodworker is making the bottom of a rectangular box with an area of 40 $square\ inches$. The width of the box is 3 $inches$ more than its length. Find the length and the width of the box.

length = x

width = $x+3$

$l \cdot w = A$

$x(x+3) = 40$

$x^2 + 3x - 40 = 0$

$(x+8)(x-5) = 0$

$x = \cancel{-8}$ $x = 5$

The length is 5 inches

The width is 8 inches

Section 7: Solve the following word problems. Solutions must include units and show all algebraic work to receive full credit. *(4 points each)*

29) James and Nathan are electricians wiring a house. It would take James 9 $hours$ if he was working alone, and it would take Nathan 18 $hours$ if he was working alone. How long will it take them working together?

Together time = x

$\frac{1}{9} + \frac{1}{18} = \frac{1}{x}$ LCD $= 18x$

$2x + x = 18$

$3x = 18$

$x = 6$

It will take 6 hrs working together

30) Andre owns a bakery that makes apple pies. He records data about his sales when he changes the price of his apple pies. He is able to model his weekly profit P as a function of the price he charges for each pie as:

$$P(x) = -2x^2 + 52x - 38$$ where x = price per pie.

What price should he charge per pie to maximize profit? What is his maximum weekly profit?

$a = -2$, $b = 52$

$x = \frac{-b}{2a} = \frac{-52}{-4} = 13$

$P(13) = -2(13)^2 + 52(13) - 38$

$-338 + 676 - 38$

$P(13) = 300$

He should charge $13 and will make $300 weekly profit.

8

Section 7: Solve the following word problems. Solutions must include units and show all algebraic work to receive full credit. *(4 points each)*

31) I launch a model rocket upwards at a speed of $112\ feet\ per\ second$. The height $h(t)$ in meters after t seconds is given by the equation:

$$h(t) = -16t^2 + 112t$$

When will the rocket hit the ground?

t = time

$h = 0, \quad 0 = -16t^2 + 112t$

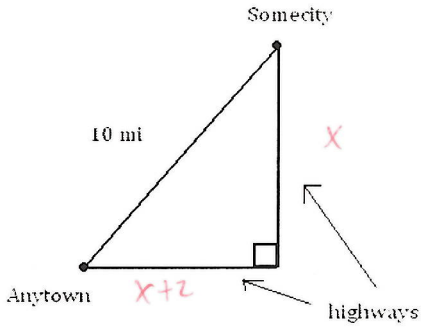
$0 = -16t(t - 7)$

$t = 0, \quad t = 7$

It will hit the ground after 7 seconds

Extra Credit: 4 possible point. NO CREDIT FOR GUESSING

Justin must drive from Anytown to Somecity. He can drive 10 miles on a rural road straight there at 40mph. Or he can drive east then north on highways at 60mph. The alternative route forms a right angle (see picture). The eastern leg is 2 miles longer than the northern leg. Which route will get him to Somecity quicker?



$(x)^2 + (x+2)^2 = 10^2$

$x^2 + x^2 + 4x + 4 = 100$

$2x^2 + 4x - 96 = 0$

$2[x^2 + 2x - 48] = 0$

$2(x+8)(x-6)$

$x = \cancel{-8} \quad x = 6$

Rural

$d = 10, \quad r = 40$

$t = \frac{10}{40} = \frac{1}{4}\ hr$

$= .25\ hr$

Highways

$d = 6 + 8 = 14, \quad r = 60$

$t = \frac{14}{60} = \frac{7}{30}\ hr$

$= .2\bar{3}\ hr$

The highways are faster

4